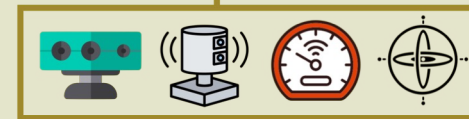
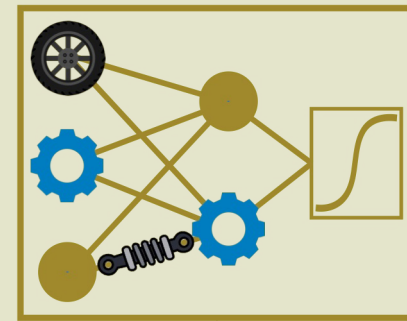


Physics-Encoded Architectures Part III: Model-Structured Learning

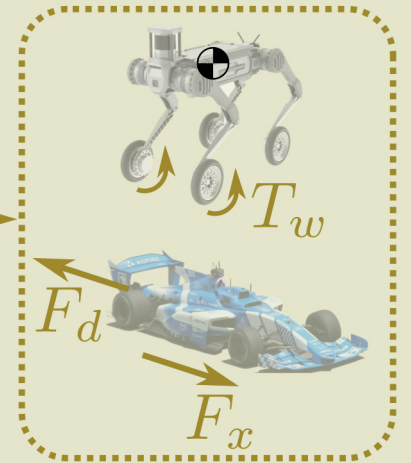
Dr. Mattia Piccinini &
Prof. Gastone Pietro Rosati Papini

University of Trento &
Technical University of Munich,
Autonomous Vehicle Systems Lab

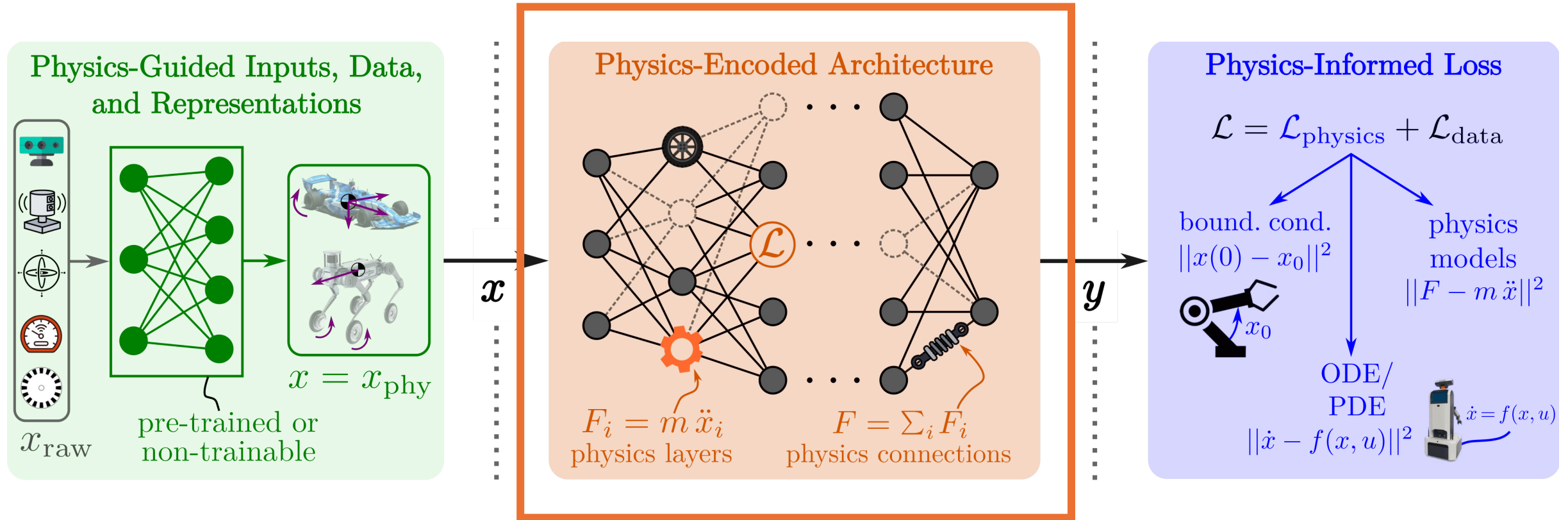
Model-Structured



*Physics-based layers,
connections, constraints*

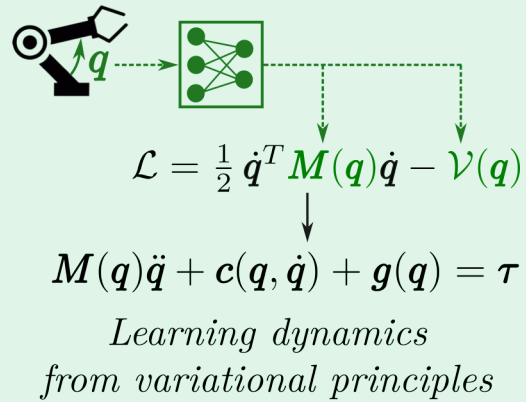


Model Structured Architectures: Positioning

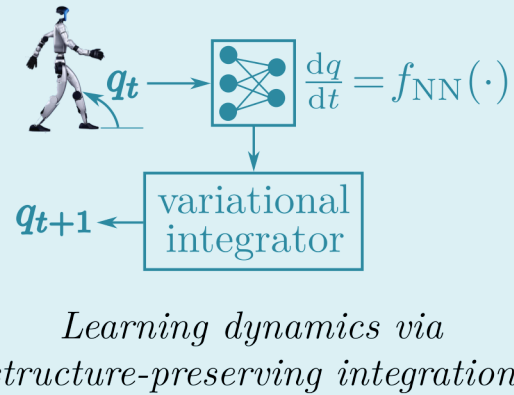


Physics-Encoded Learning Architectures: Multiple Classes

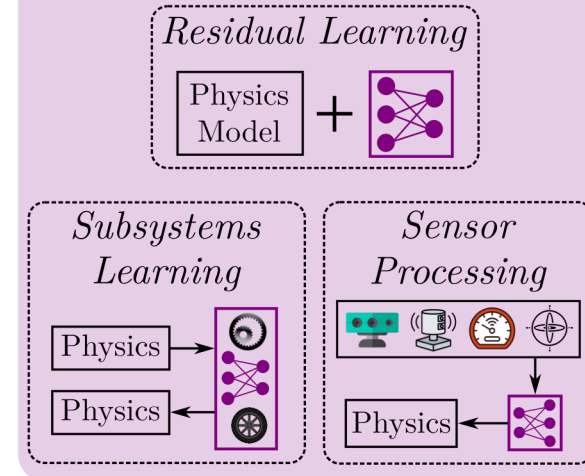
Lagrangian & Hamiltonian Approaches



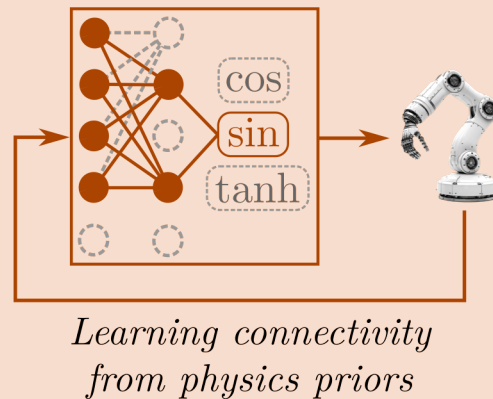
NODE & Variational Integrator Networks



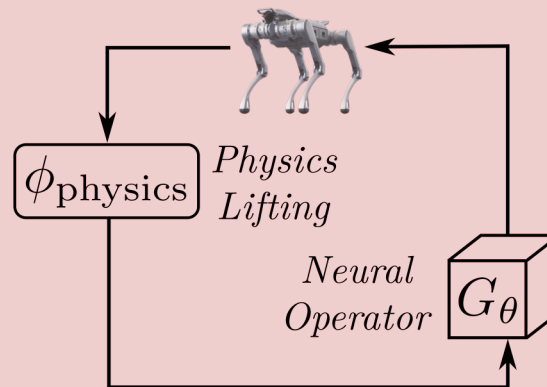
Hybrid Physics-Neural



Physics-Encoded Topology Learning

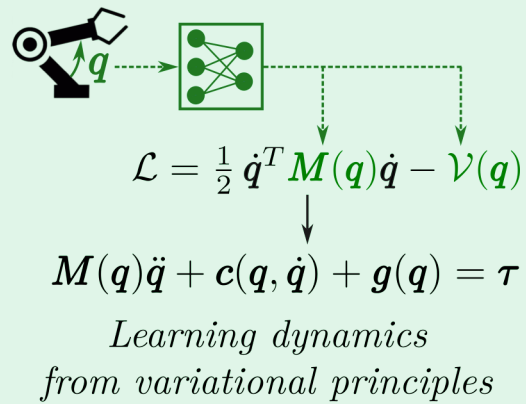


Physics-Encoded Neural Operators

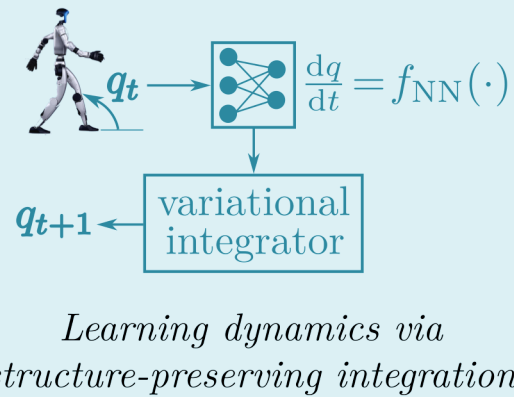


Physics-Encoded Learning Architectures: Multiple Classes

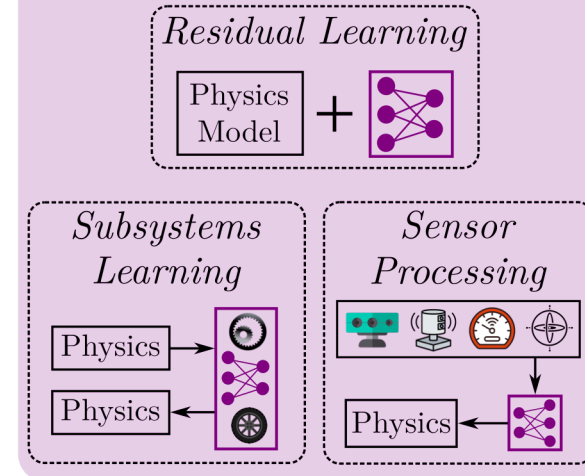
Lagrangian & Hamiltonian Approaches



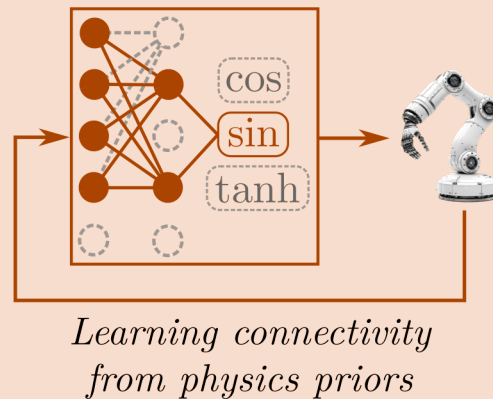
NODE & Variational Integrator Networks



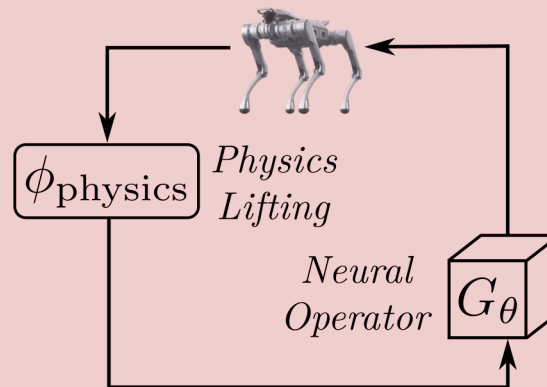
Hybrid Physics-Neural



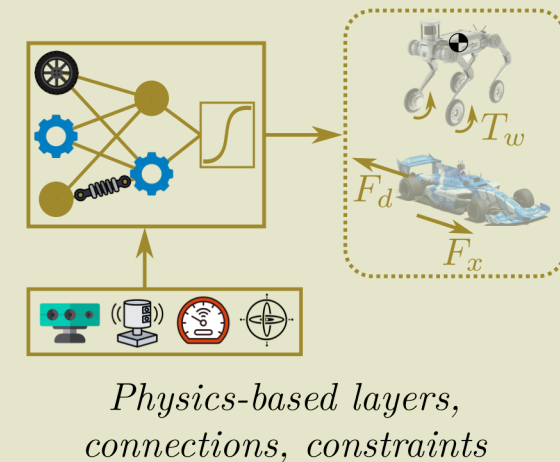
Physics-Encoded Topology Learning



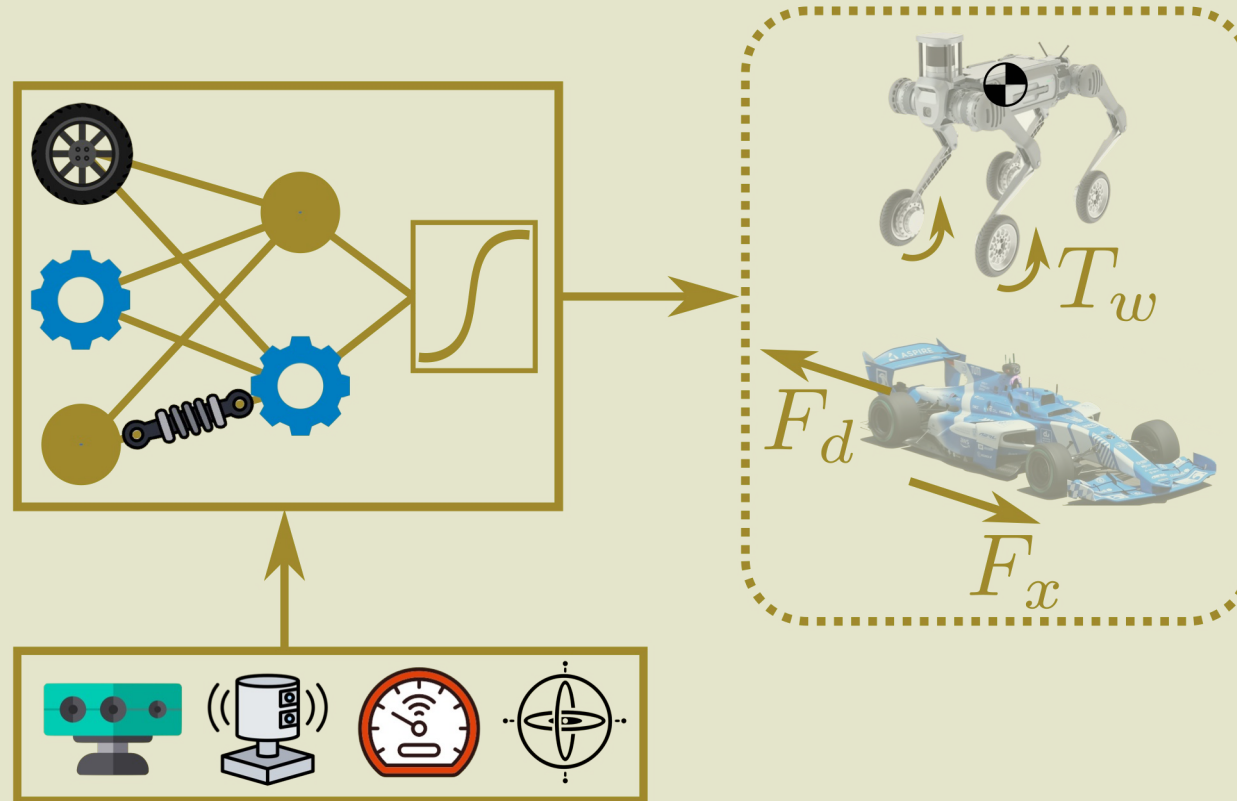
Physics-Encoded Neural Operators



Model-Structured



Model-Structured



*Physics-based layers,
connections, constraints*

Taxonomy

The term «**Model-Structured Neural Network**» (**MS-NN**) has recently been introduced by papers of our labs (2025-2026) and aims to cover a broad class of approaches

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Intelligent Transportation
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Digital Object Identifier 10.1109/OJITS.2025.3563347

A Road Friction-Aware Anti-Lock Braking System Based on Model-Structured Neural Networks

MATTIA PICCININI^{1,2} (Member, IEEE), MATTEO ZUMERLE³, JOHANNES BETZ^{1,2} (Member, IEEE), AND GASTONE PIETRO ROSATI PAPINI³ (Member, IEEE)

2025 IEEE 28th International Conference on Intelligent Transportation Systems (ITSC)
November 18-21, 2025. Gold Coast, Australia

Model-Structured Neural Networks to Control the Steering Dynamics of Autonomous Race Cars

Mattia Piccinini^{1*}, Aniello MungIELLO^{2*}, Georg Jank³, Gastone Pietro Rosati Papini⁴,
Francesco Biral⁴, Johannes Betz¹

 IEEE Open Journal of
Intelligent Transportation
Systems

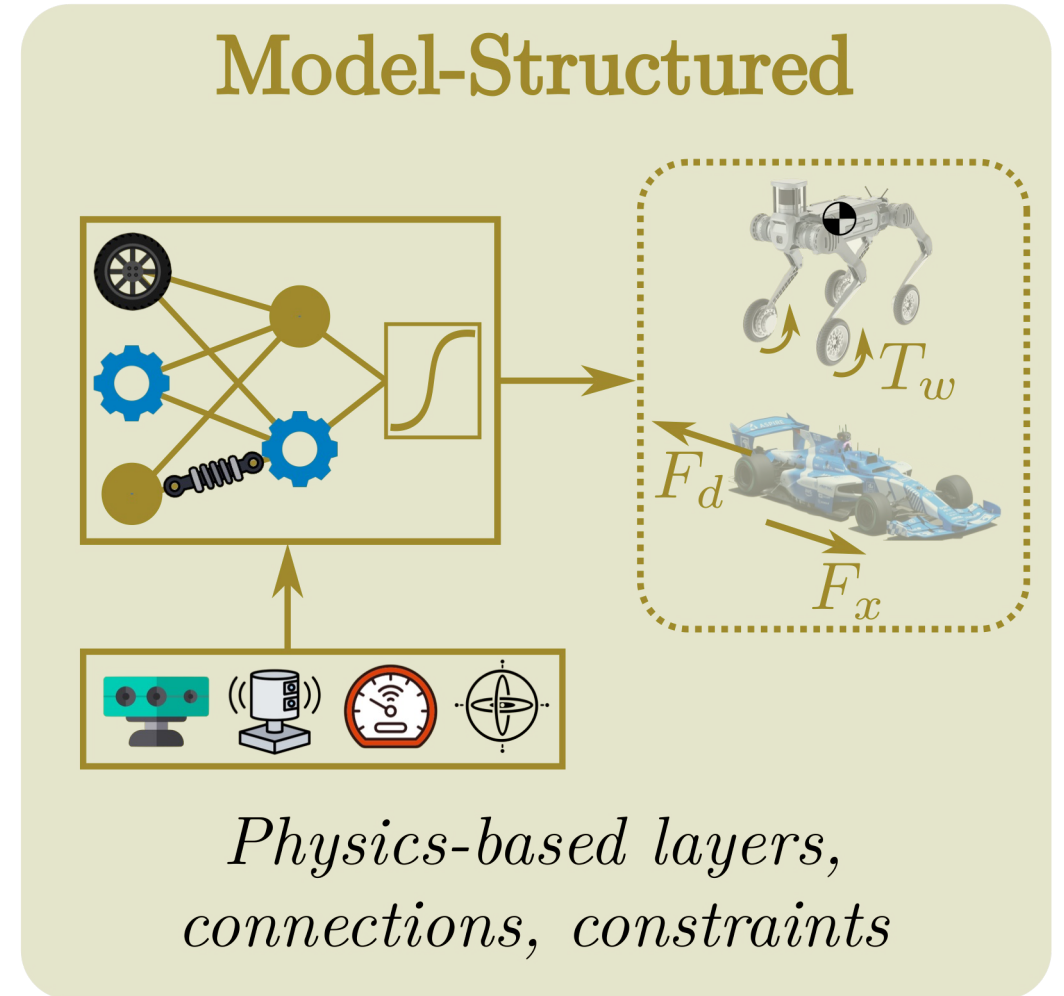
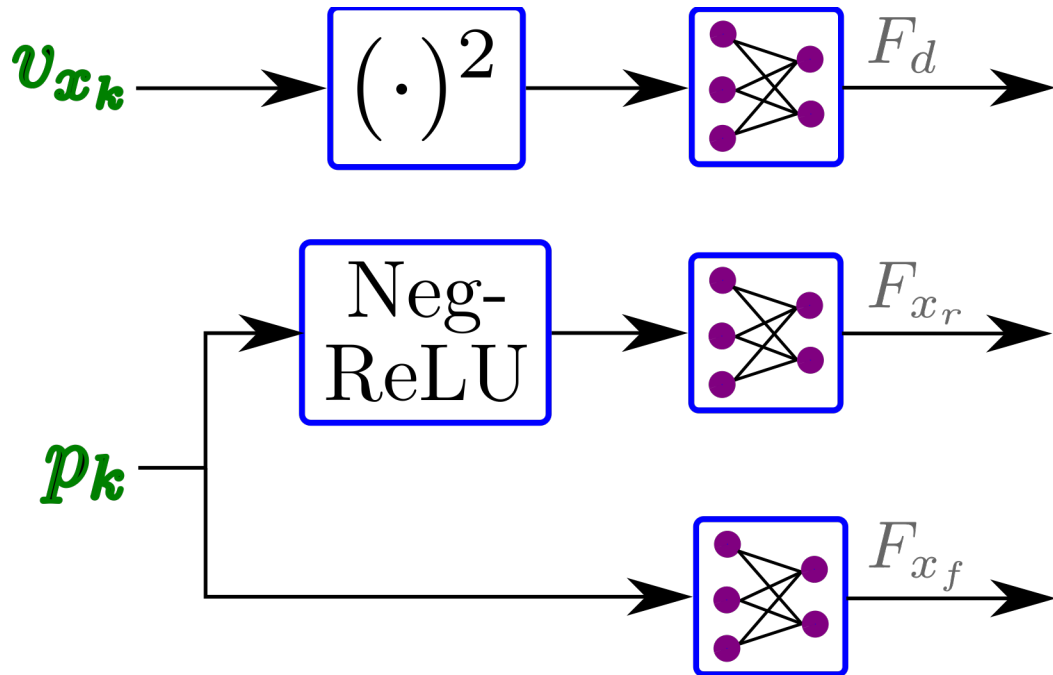
Received 22 December 2025; revised 27 March 2026; accepted 14 April 2026. Date of publication 17 April 2026; date of current version 4 May 2026.
Digital Object Identifier 10.1109/OJITS.2026.3685078

Vehicle Dynamics Learning From Physics Priors With Model-Structured Neural Networks

ANIELLO MUNGIELLO¹ (Graduate Student Member, IEEE), FELIX JAHNCKE² (Graduate Student Member, IEEE), STEFANIA SANTINI¹, JOHANNES BETZ³ (Member, IEEE), GASTONE PIETRO ROSATI PAPINI⁴ (Member, IEEE), AND MATTIA PICCININI² (Member, IEEE)

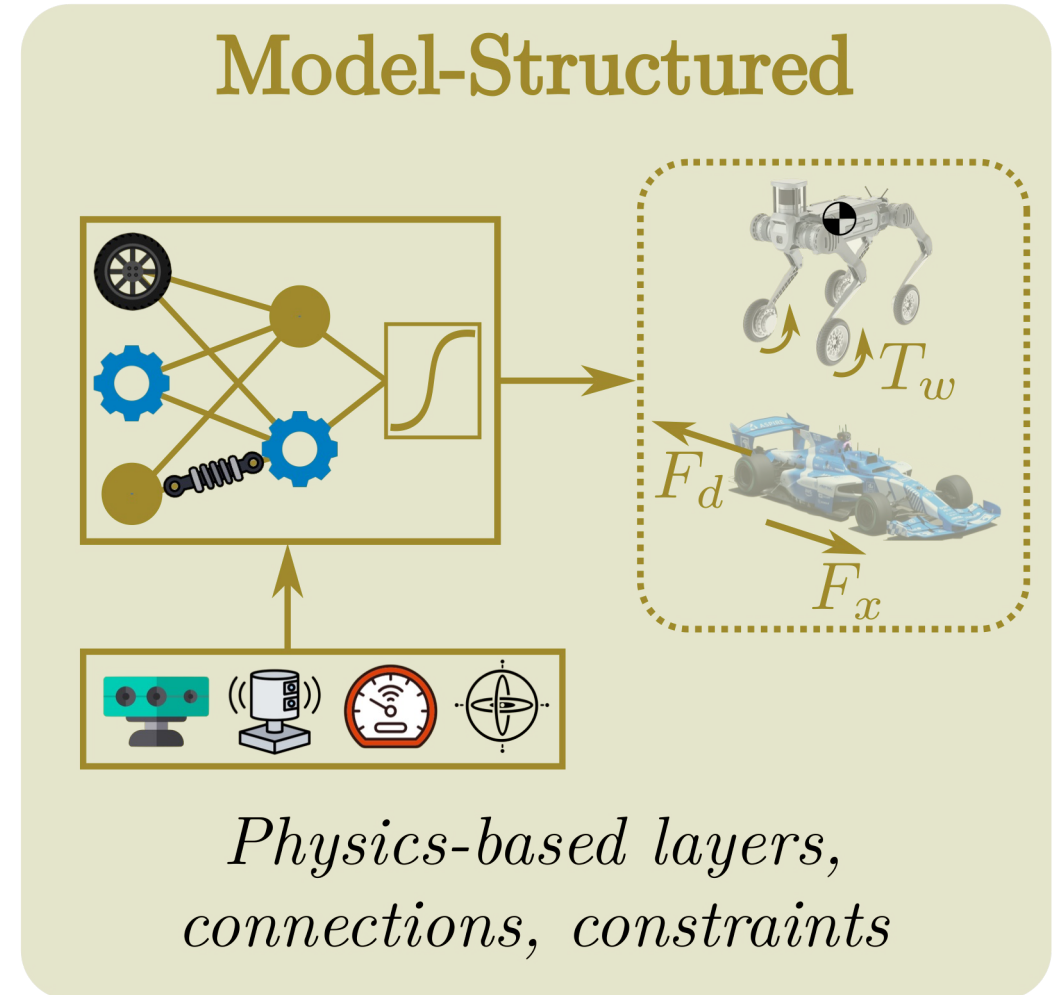
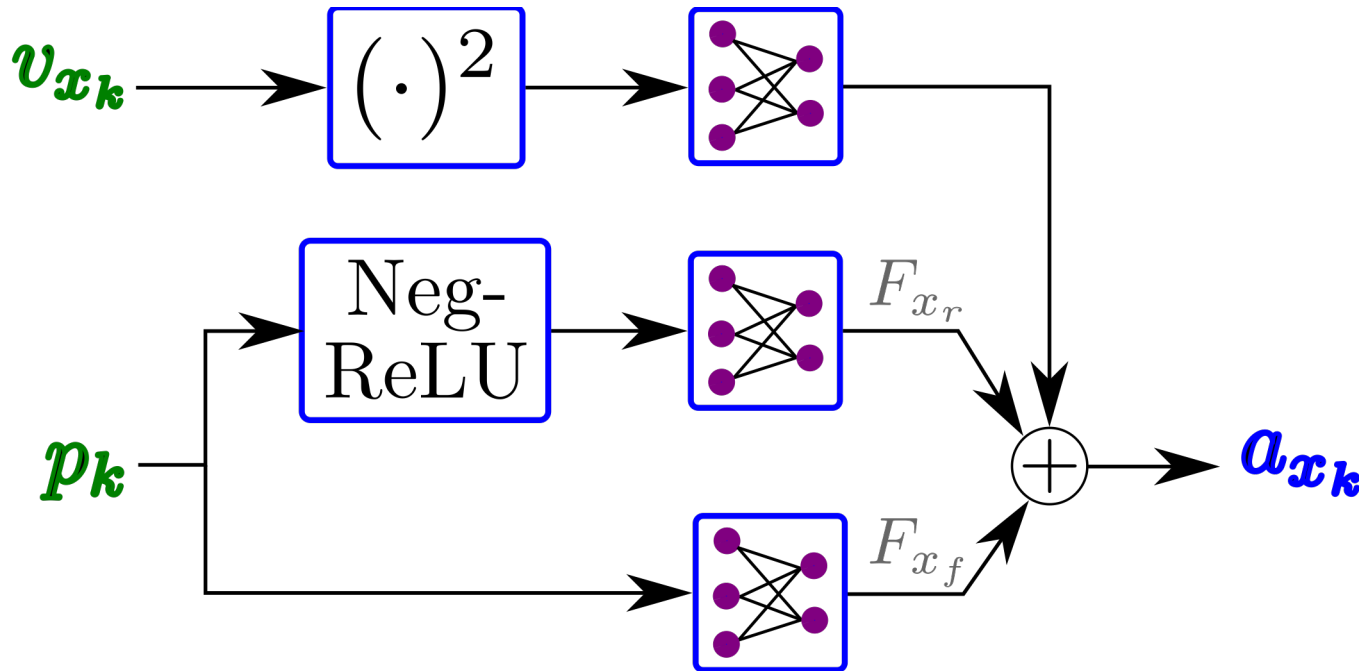
Model Structured Architectures: General Principles

- **Physics-based layers:** encoding custom physics equations / principles



Model Structured Architectures: General Principles

- **Physics-based layers:** encoding custom physics equations / principles
- **Physics-based connections:** superposition of forces, symmetries, ...

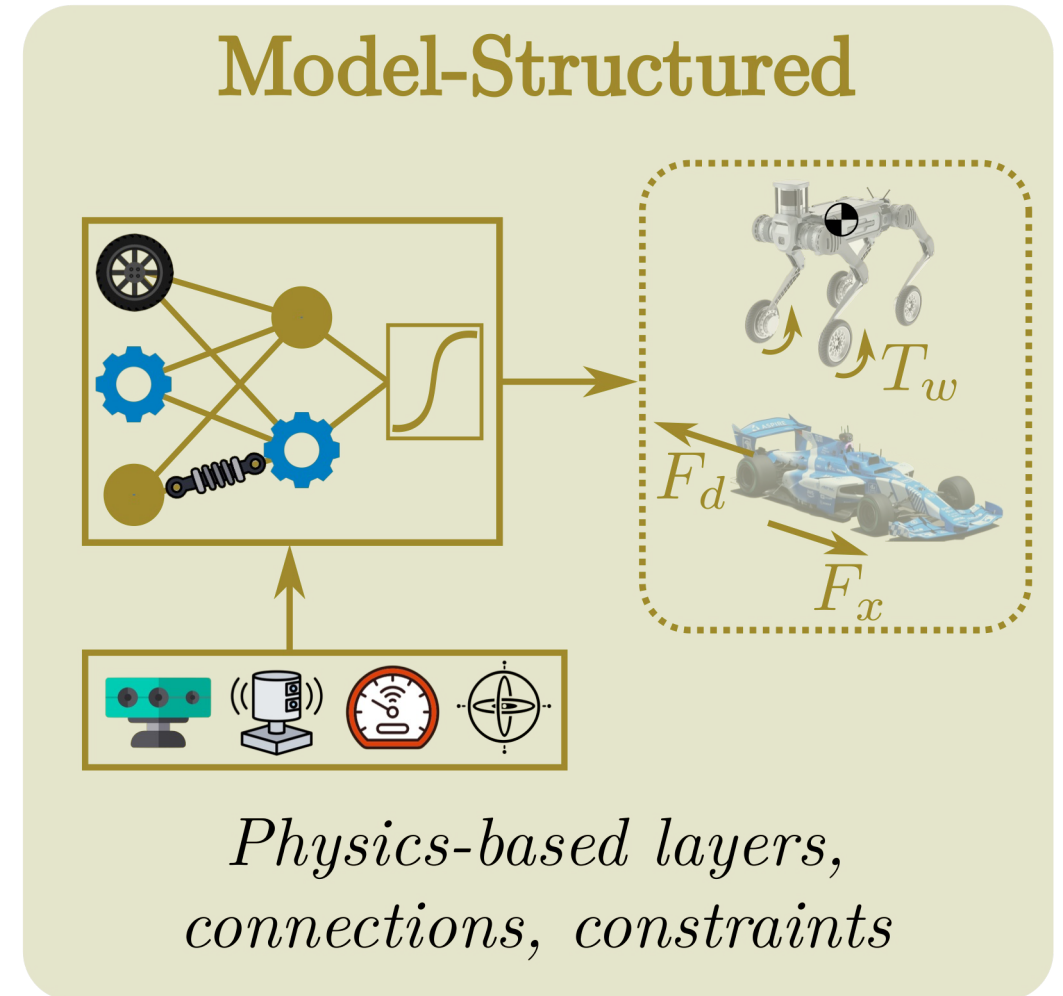


Model Structured Architectures: General Principles

- **Physics-based layers:** encoding custom physics equations / principles
- **Physics-based connections:** superposition of forces, symmetries, ...
- **Physics-based constraints:** Constrain parameters to have physically meaningful values
 - Note: Only „**architectural**“ constraints, no loss function regularization terms!
 - Example from Chrosniak et al., 2024

$$\hat{\Phi}_u = \sigma(z) \cdot (\bar{\Phi}_u - \underline{\Phi}_u) + \underline{\Phi}_u$$

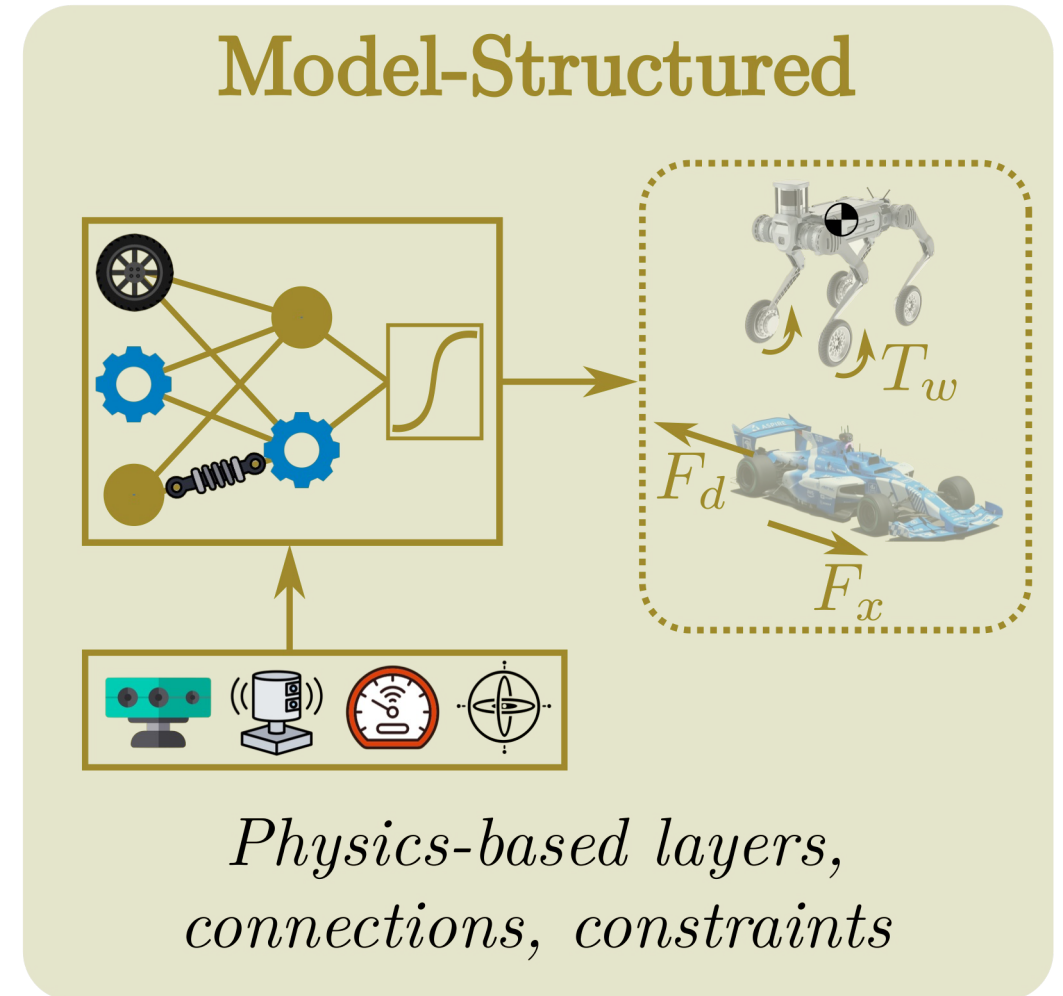
σ = sigmoid fun, $\bar{\Phi}_u$ = upper bound, $\underline{\Phi}_u$ = lower bound



Model Structured Architectures: General Principles

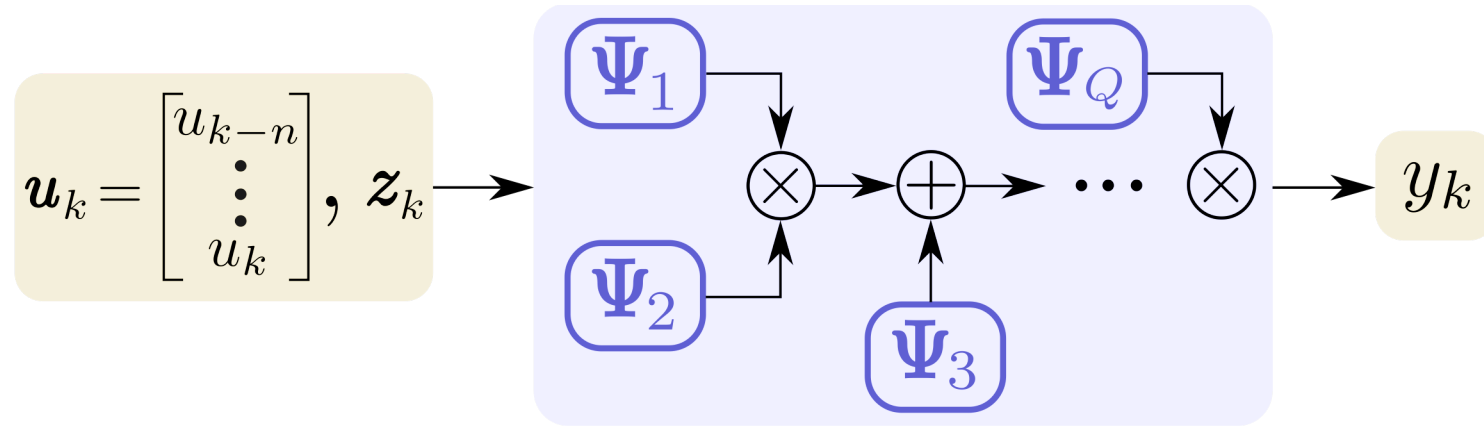
- **Physics-based layers:** encoding custom physics equations / principles
- **Physics-based connections:** superposition of forces, symmetries, ...
- **Physics-based constraints:** Constrain parameters to have physically meaningful values

Practical guideline: start from an (imperfect) physical model and *neuralize* its architecture



Model Structured Architectures: **Proposed Design**

General approach for **dynamics learning of physical systems** via
Neural Compositional Modules (NCMs) Ψ



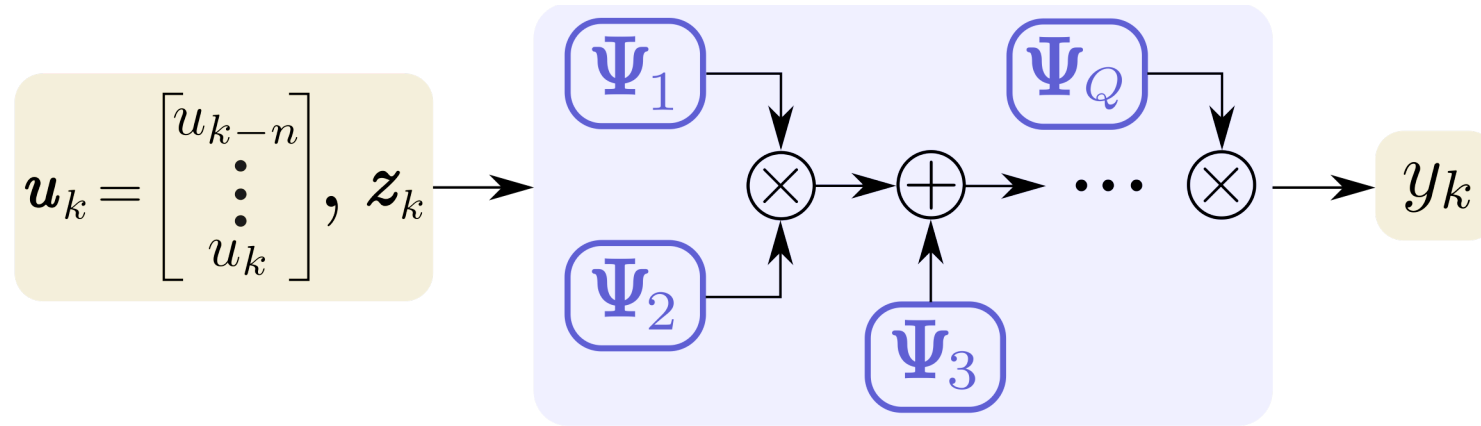
y_k = system output, $\mathbf{u}_k = [u_{k-n}, \dots, u_k]$ = history of inputs, $\mathbf{z}_k$ = scheduling vars

$$y_k = \mathcal{F}(\Psi_1, \Psi_2, \dots, \Psi_Q), \quad \Psi_i = \Psi_i(\mathbf{u}_k, \mathbf{z}_k), \quad i \in \{1, \dots, Q\}$$

Where $\mathcal{F} \in \mathcal{C}(O)$, $O = \{+, \times\}$ denotes the set of all compositions generated by sums and multiplications

Model Structured Architectures: **Proposed Design**

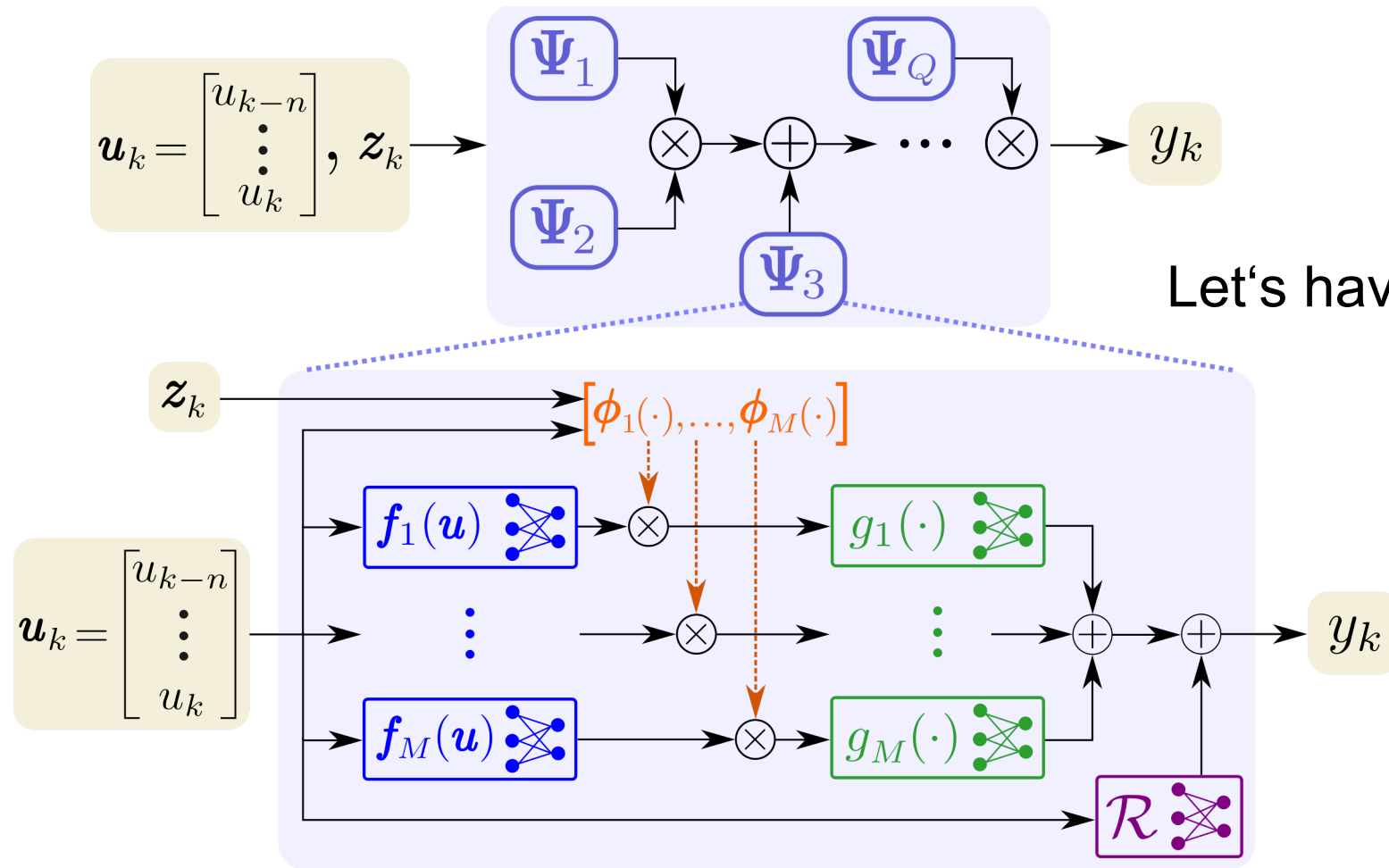
General approach for **dynamics learning of physical systems** via
Neural Compositional Modules (NCMs) Ψ



How to design an **NCM Ψ** ?

Model Structured Architectures: **Proposed Design**

General approach for **dynamics learning of physical systems** via
Neural Compositional Modules (NCMs) Ψ



Let's have a closer look!

Neural Compositional Modules: Input / Output Representation

Consider a physical system described by a nonlinear time-invariant state-space model

$$\begin{aligned}x_{k+1} &= f(x_k, u_k) \\ y_k &= h(x_k, u_k)\end{aligned}$$



Substitute x_k from the first into the second eq.

$$\begin{aligned}y_k = h(x_k, u_k) &= h(f(x_{k-1}, u_{k-1}), u_k) \\ &= \dots = \mathcal{G}(u_k, u_{k-1}, \dots, u_{k-n}, x_{k-n})\end{aligned}$$



$$y_k = \mathcal{G}(u_k, u_{k-1}, \dots, u_{k-n}, \cancel{x_{k-n}})$$

If the past history of inputs (n) is „*long enough*“, the effect of the initial state x_{k-n} is negligible (the free response vanishes, and the input-forced response dominates)

Neural Compositional Modules: Input / Output Representation

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$$y_k = \mathcal{G}(u_k, u_{k-1}, \dots, u_{k-n})$$

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Neural Compositional Modules: Input / Output Representation

$$y_k = \mathcal{G}(u_k, u_{k-1}, \dots, u_{k-n})$$

Also assume that we can have different **parametrizations** of $\mathcal{G}(\cdot)$ in **different operating regimes** (with partial or no overlap among the regimes), with **scheduling variables** z_k



Full NCM:

$$\underbrace{y_k}_{\text{output}} = \Psi \left(\underbrace{u_k, u_{k-1}, \dots, u_{k-n}}_{\text{history of inputs}}, \underbrace{z_k}_{\text{scheduling vars}} \right)$$

Questions & Discussion



Neural Compositional Modules: Internal Architecture

$$y_k = \Psi(u_k, u_{k-1}, \dots, u_{k-n}, z_k)$$

If the physical system is **linear**: $y_k = (\Gamma * \mathbf{u})_k = \sum_{i=0}^{\infty} \Gamma_i u_{k-i}$

- Γ is the **discrete-time transfer function** (= impulse response) of the system

Neural Compositional Modules: Internal Architecture

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- Γ is the **discrete-time transfer function** (= impulse response) of the system
- **Infinite Impulse Response (IIR)** system

Neural Compositional Modules: Internal Architecture

$$y_k = \Psi(u_k, u_{k-1}, \dots, u_{k-n}, z_k)$$

If the physical system is **linear**: $y_k = (\Gamma * \mathbf{u})_k \approx \sum_{i=0}^n \Gamma_i u_{k-i}$

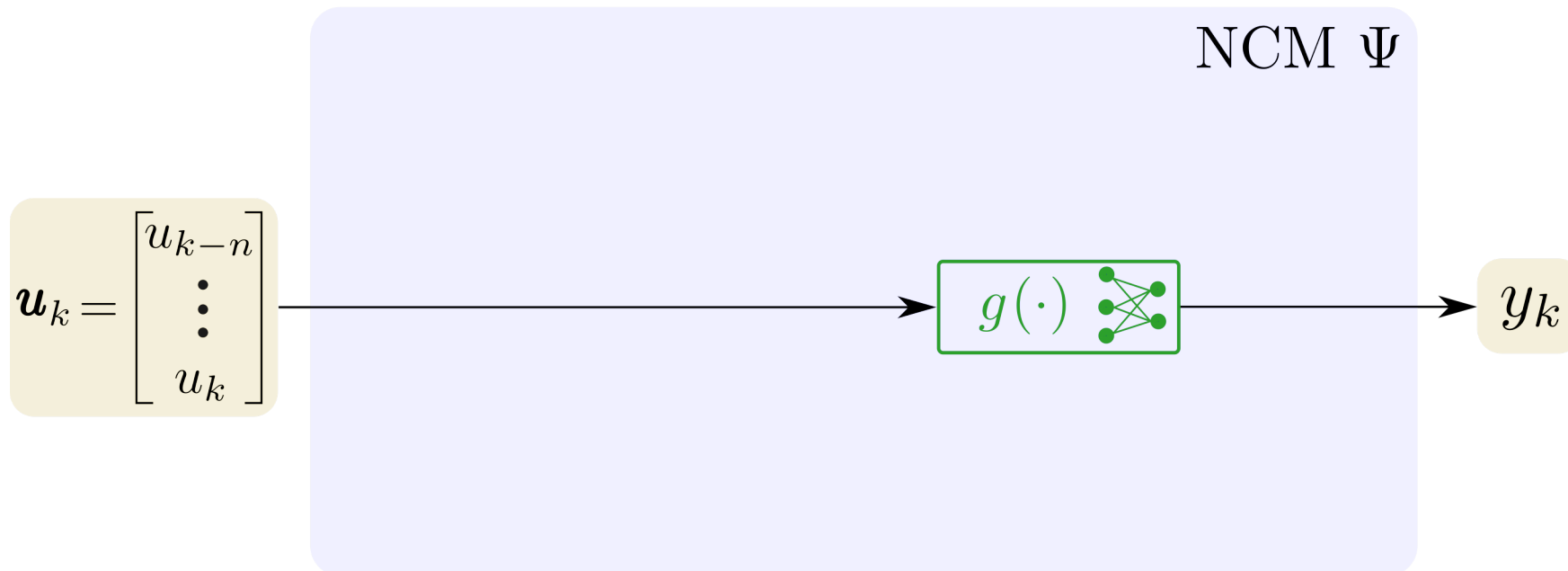
- Γ is the **discrete-time transfer function** (= impulse response) of the system
- **Finite Impulse Response (FIR)** approximation

Neural Compositional Modules: Internal Architecture

$$y_k = \Psi(u_k, u_{k-1}, \dots, u_{k-n}, z_k)$$

If the physical system is **linear**: $y_k = (\Gamma * \mathbf{u})_k \approx \sum_{i=0}^n \Gamma_i u_{k-i} = g(\mathbf{u}_k)$

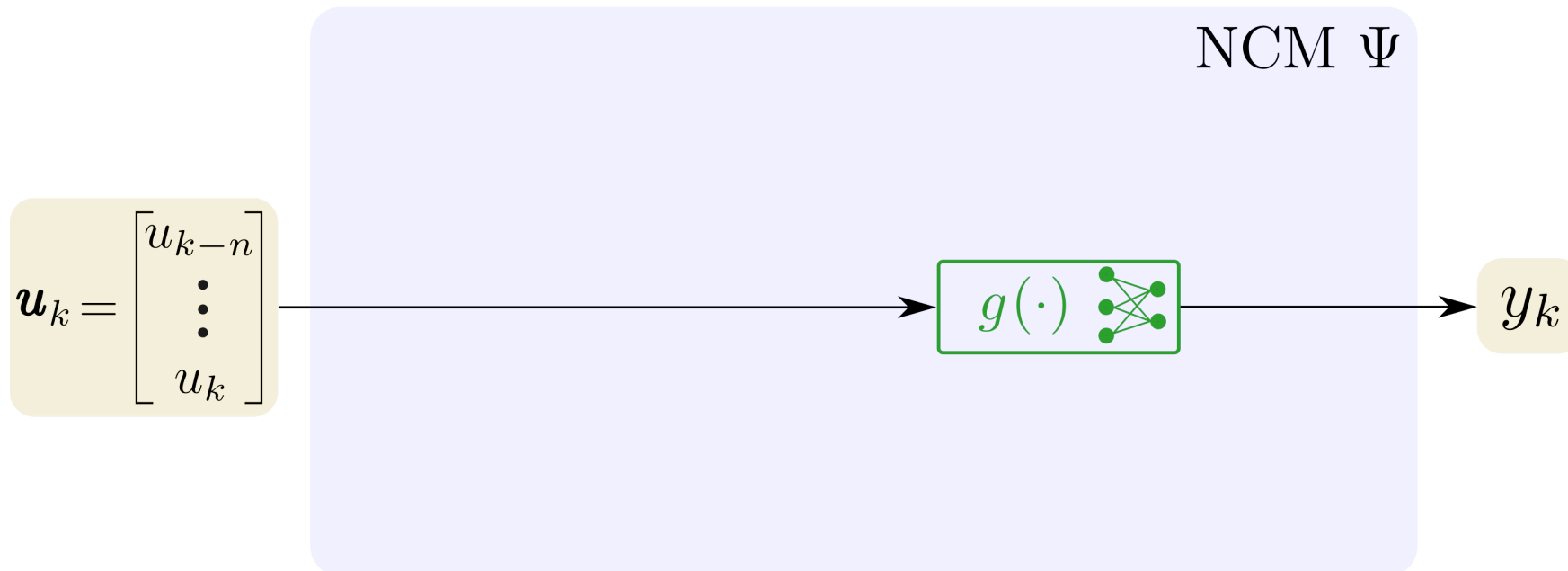
- Γ is the **discrete-time transfer function** (= impulse response) of the system
- **Finite Impulse Response (FIR)** approximation = **1 linear fully connected layer** g whose weights are the impulse response coefficients



Neural Compositional Modules: Internal Architecture

$$y_k = \Psi(u_k, u_{k-1}, \dots, u_{k-n}, z_k)$$

If the physical system is **linear**: $y_k = \sum_{i=0}^n \Gamma_i u_{k-i} = g(\mathbf{u}_k)$

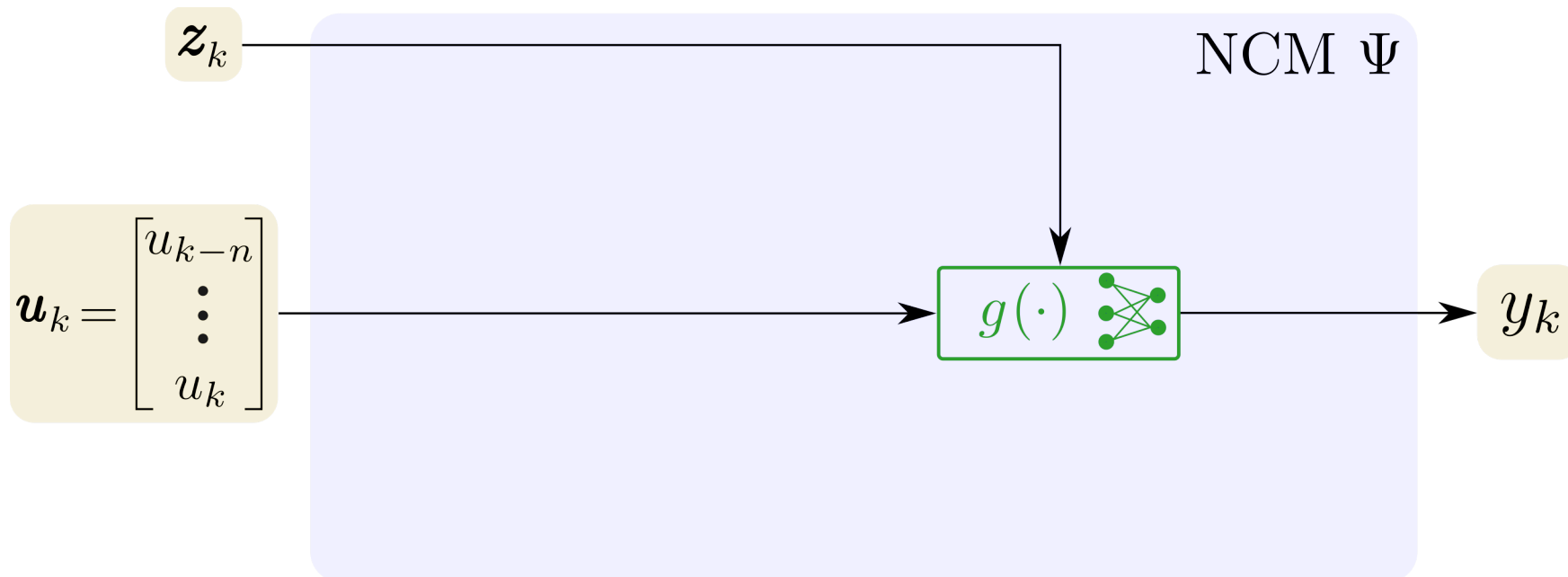


Neural Compositional Modules: Internal Architecture

$$y_k = \Psi(u_k, u_{k-1}, \dots, u_{k-n}, z_k)$$

If the physical system is **locally linear**: $y_k = \sum_{i=0}^n \Gamma_i(\mathbf{z}_k) u_{k-i} = g(\mathbf{u}_k, \mathbf{z}_k)$

- $\mathbf{z}_k$ is a (set of) discrete or continuous scheduling variables. For example, $\mathbf{u}_k \subseteq \mathbf{z}_k$
- We propose to have different modules $g_j(\mathbf{u}_k)$ activated locally by $\phi_j(\mathbf{z}_k)$, $j \in \{1, \dots, M\}$

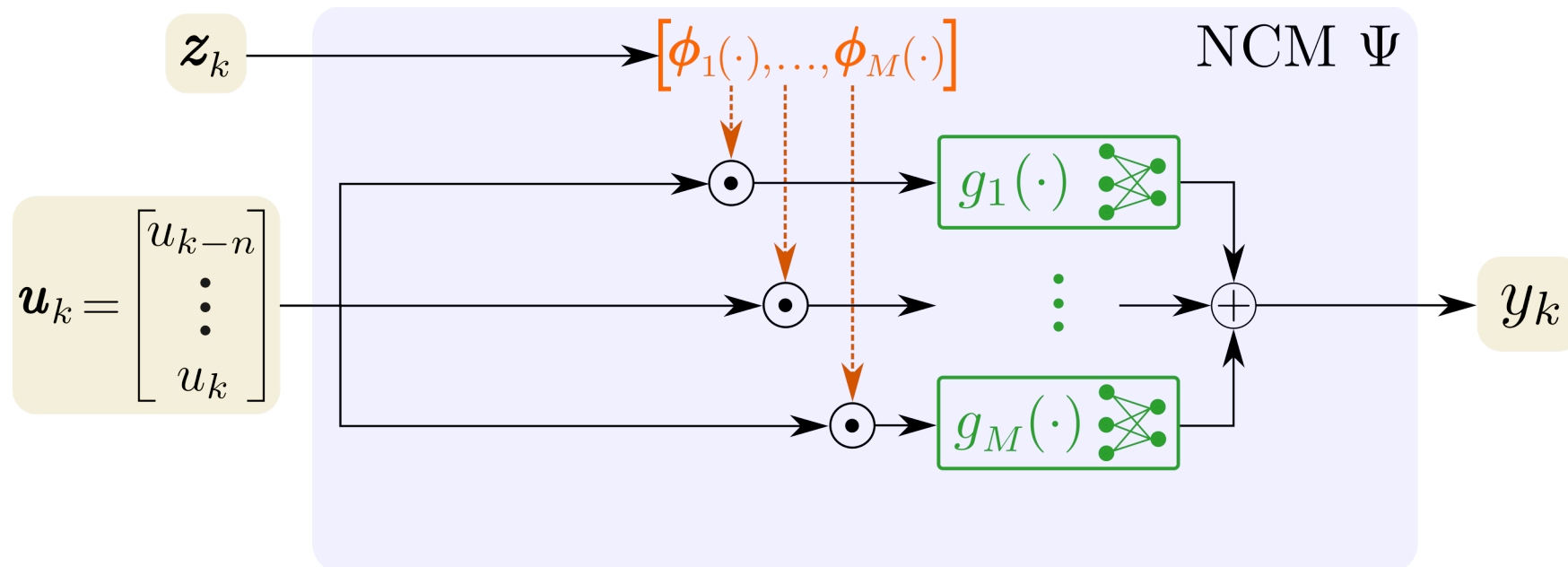


Neural Compositional Modules: Internal Architecture

$$y_k = \Psi(u_k, u_{k-1}, \dots, u_{k-n}, z_k)$$

If the physical system is **locally linear**: $y_k = \sum_{j=1}^M [u_k \odot \phi_j(z_k)] \cdot g_j$

- z_k is a (set of) discrete or continuous scheduling variables. For example, $u_k \subseteq z_k$
- We propose to have different modules $g_j(u_k)$ activated locally by $\phi_j(z_k)$, $j \in \{1, \dots, M\}$
- $\odot$ is the element-wise product

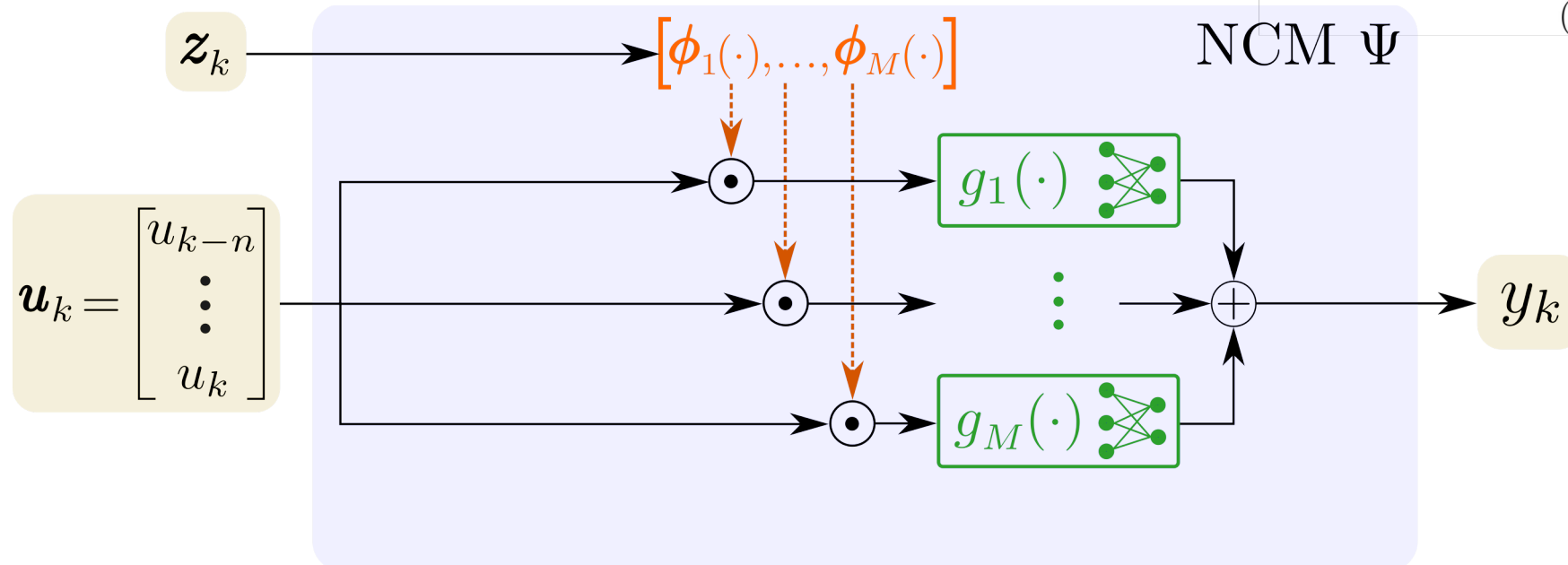
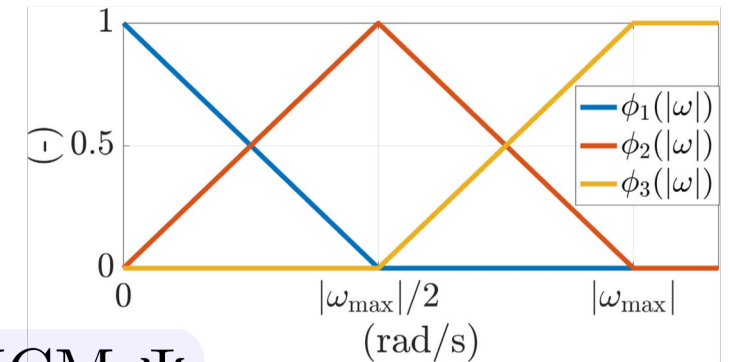


Neural Compositional Modules: Internal Architecture

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If the physical system is **locally linear**: $y_k = \sum_{j=1}^M [u_k \odot \phi_j(z_k)] \cdot g_j$

- $\phi_j(z_k)$ may be trainable or frozen
- Common choices: triangular, rectangular, Gaussian
- $\sum_{j=1}^M \phi_j(z_k) = 1, \quad \phi_j(z_k) \geq 0 \quad \forall z_k$



Neural Compositional Modules: Internal Architecture

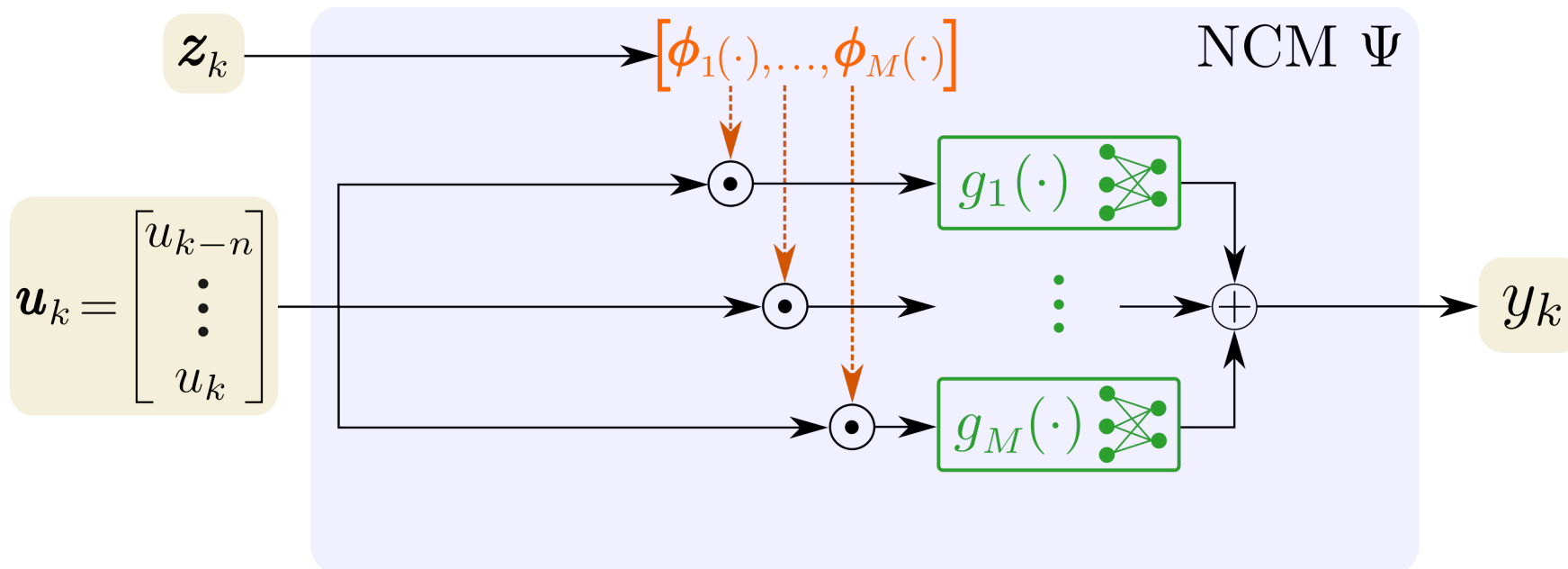
$$y_k = \Psi(u_k, u_{k-1}, \dots, u_{k-n}, z_k)$$

If the physical system is **locally linear** : $y_k = \sum_{j=1}^M [u_k \odot \phi_j(z_k)] \cdot g_j$

$u_k = [u_{k-n}, \dots, u_k]$: past history of inputs

$\phi_j = [\phi_{j_1}, \dots, \phi_{j_{n+1}}]$: regime-dependent activation functions

$g_j = [g_{j_1}, \dots, g_{j_{n+1}}]^T$: neural FIR layer (fully connected layer)

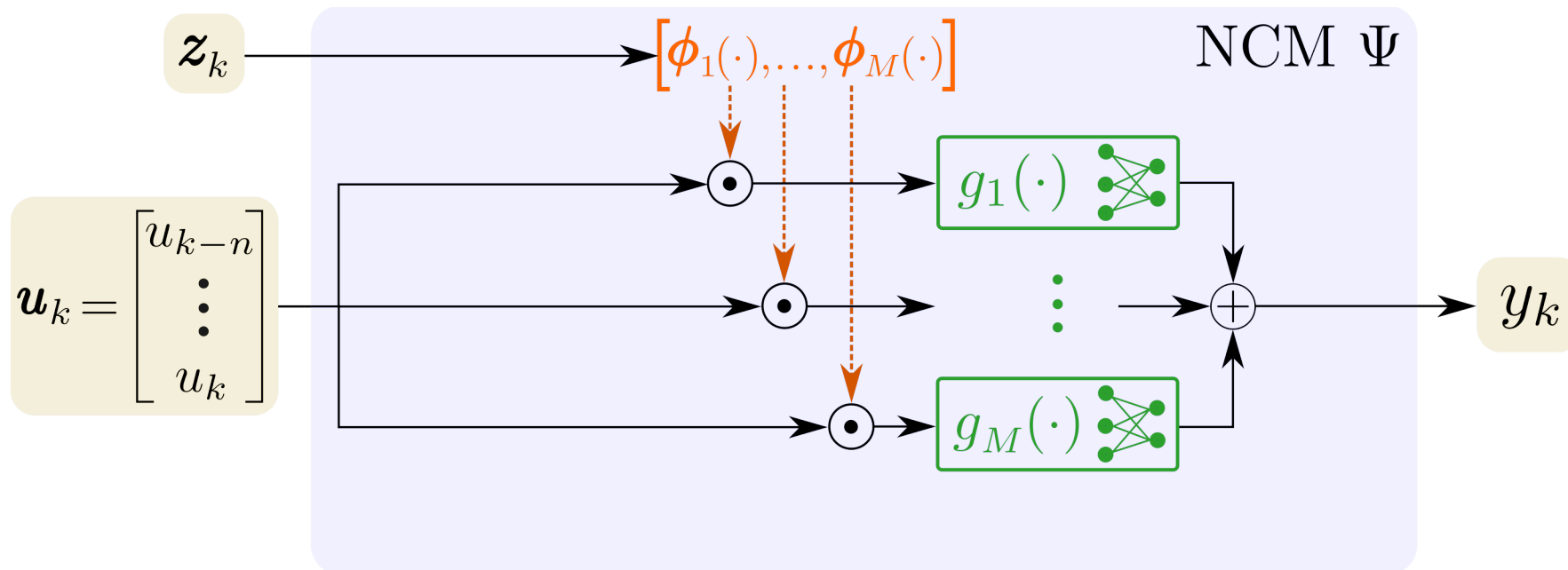


Neural Compositional Modules: Internal Architecture

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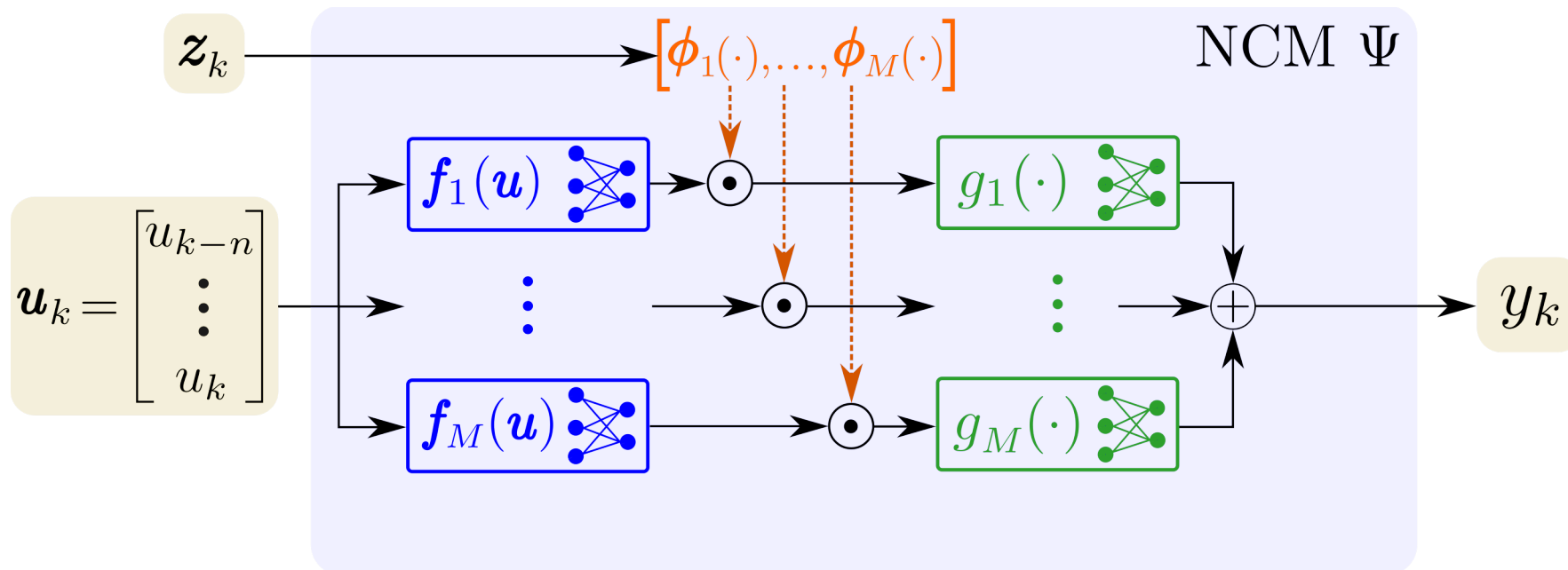


Neural Compositional Modules: Internal Architecture

$$y_k = \Psi(u_k, u_{k-1}, \dots, u_{k-n}, z_k)$$

If the physical system is **locally linear + input nonlinearities** :

$$y_k = \sum_{j=1}^M [\mathbf{f}_j(\mathbf{u}_k) \odot \boldsymbol{\phi}_j(\mathbf{z}_k)] \cdot \mathbf{g}_j \quad \text{How to design } \mathbf{f}_j(\cdot)?$$

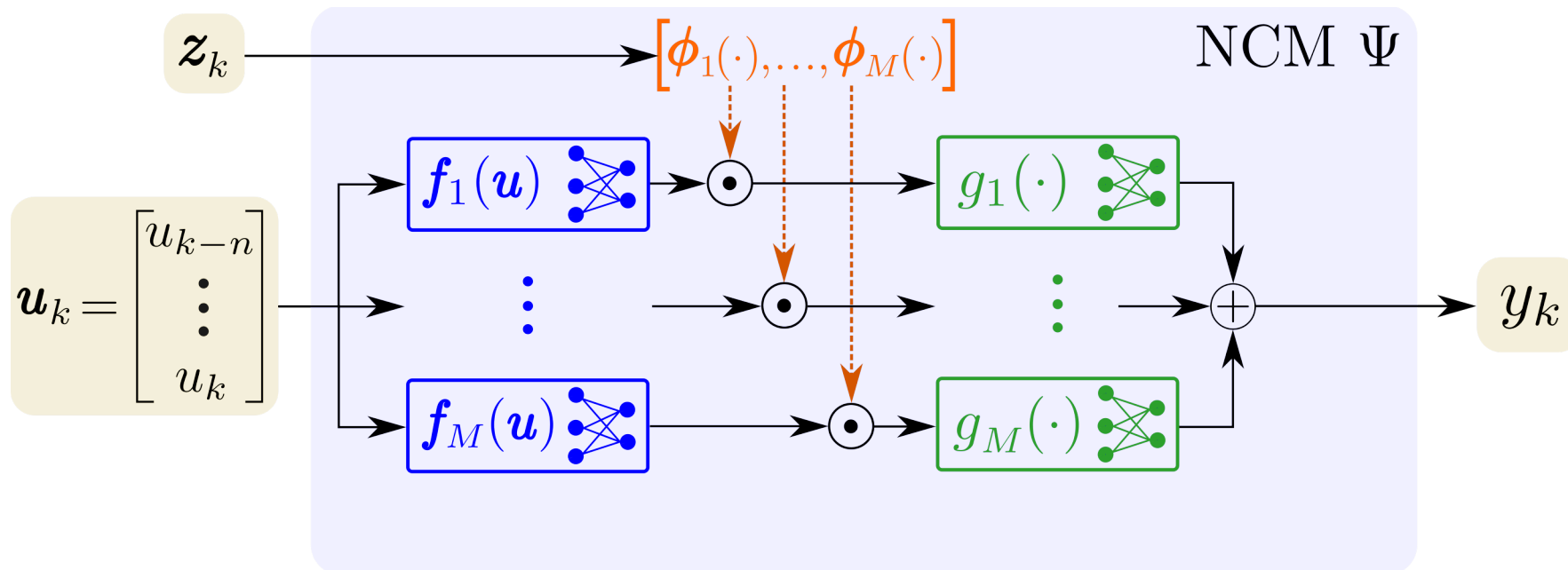


Neural Compositional Modules: Internal Architecture

$$y_k = \sum_{j=1}^M [\mathbf{f}_j(\mathbf{u}_k) \odot \boldsymbol{\phi}_j(\mathbf{z}_k)] \cdot \mathbf{g}_j$$

$\mathbf{f}(\mathbf{u}_k)$ can encode **prior physics** / domain knowledge (if available):

- Nonlinear input mappings or transformations
- May contain analytical + neural relations $\rightarrow$ designer / engineer expertise required



Neural Compositional Modules: Internal Architecture

$$y_k = \sum_{j=1}^M [\mathbf{f}_j(\mathbf{u}_k) \odot \boldsymbol{\phi}_j(\mathbf{z}_k)] \cdot \mathbf{g}_j$$

What if only **limited physics priors** are available?

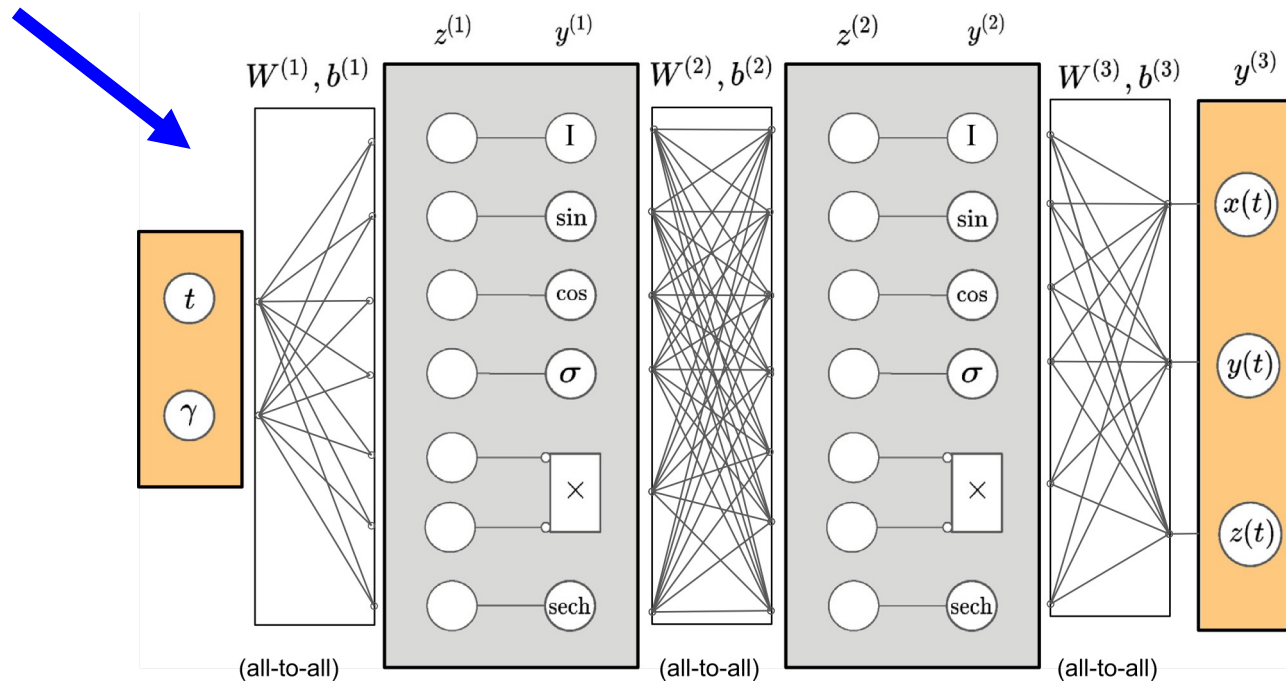
- For example, the dynamics of a robot manipulator are a **combination** of **trigonometric** functions, but the exact form is **unknown**

Neural Compositional Modules: Internal Architecture

$$y_k = \sum_{j=1}^M [\mathbf{f}_j(\mathbf{u}_k) \odot \boldsymbol{\phi}_j(\mathbf{z}_k)] \cdot \mathbf{g}_j$$

What if only **limited physics priors** are available?

- In this case, $\mathbf{f}(\mathbf{u}_k)$ can learn to combine trigonometric functions with **Equation Learning**



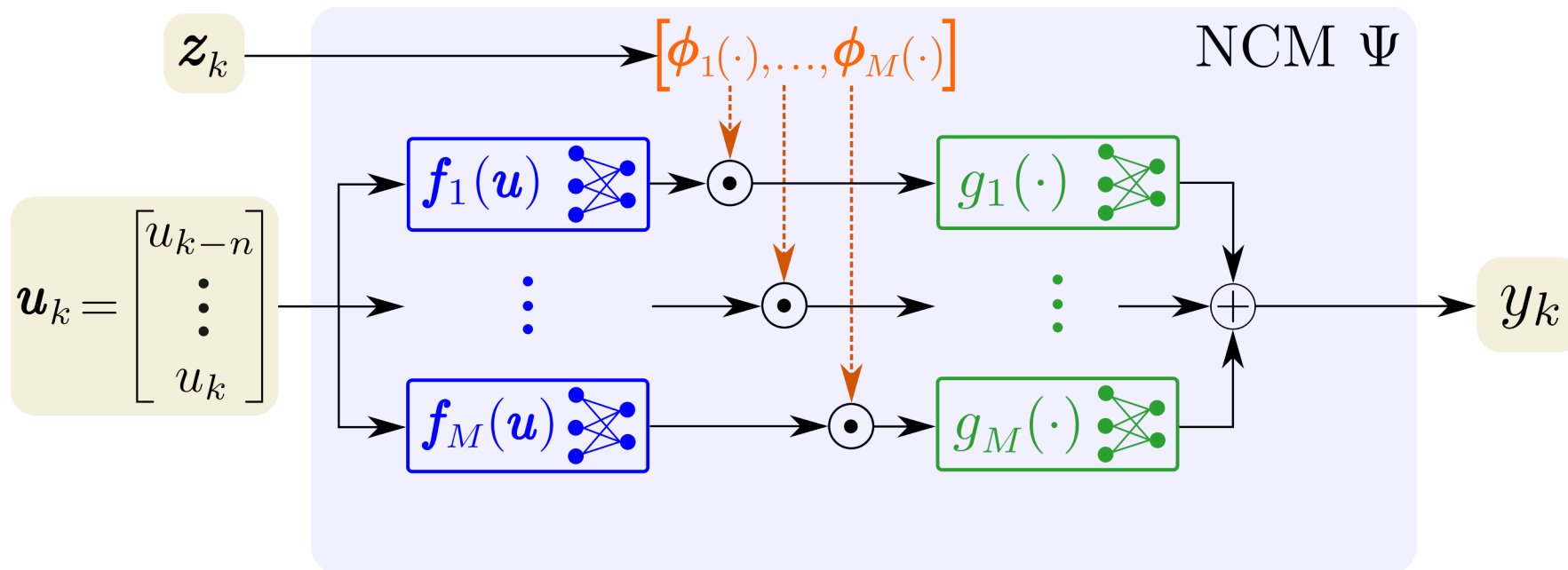
Neural Compositional Modules: Internal Architecture

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What if **no physics priors** are available?

- $\mathbf{f}_j(\mathbf{u}_k)$ can be a **learnable Taylor expansion** of the unknown nonlinear relation

$$\mathbf{f}_j(\mathbf{u}_k) = \mathbf{a}_0 + \mathbf{a}_1 \mathbf{u}_k + \mathbf{a}_2 \mathbf{u}_k^2 \quad (\text{2nd order Taylor with learnable } \mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2)$$

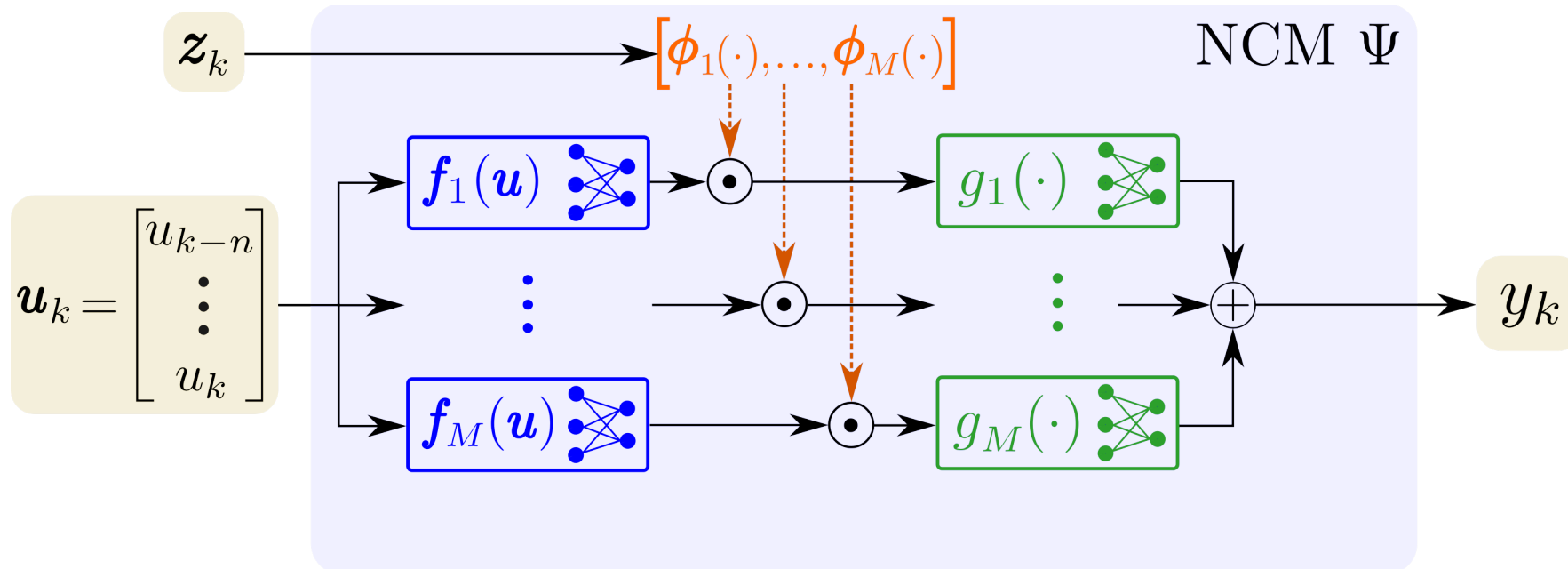


Neural Compositional Modules: Internal Architecture

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What if **no physics priors** are available?

- $\mathbf{f}_j(\mathbf{u}_k)$ can be a **learnable Taylor expansion** of the unknown nonlinear relation
- ... Or a more general-purpose machine learning model

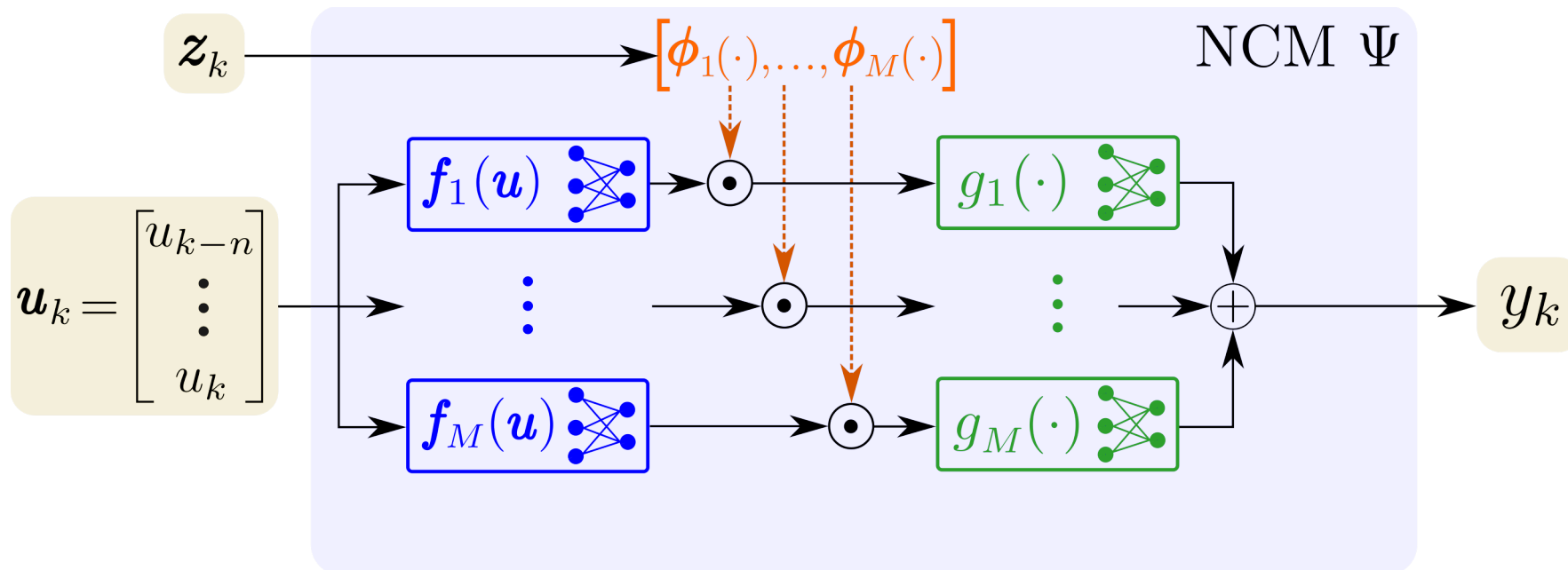


Neural Compositional Modules: Internal Architecture

$$y_k = \Psi(u_k, u_{k-1}, \dots, u_{k-n}, z_k)$$

If the physical system is **locally linear + input nonlinearities** :

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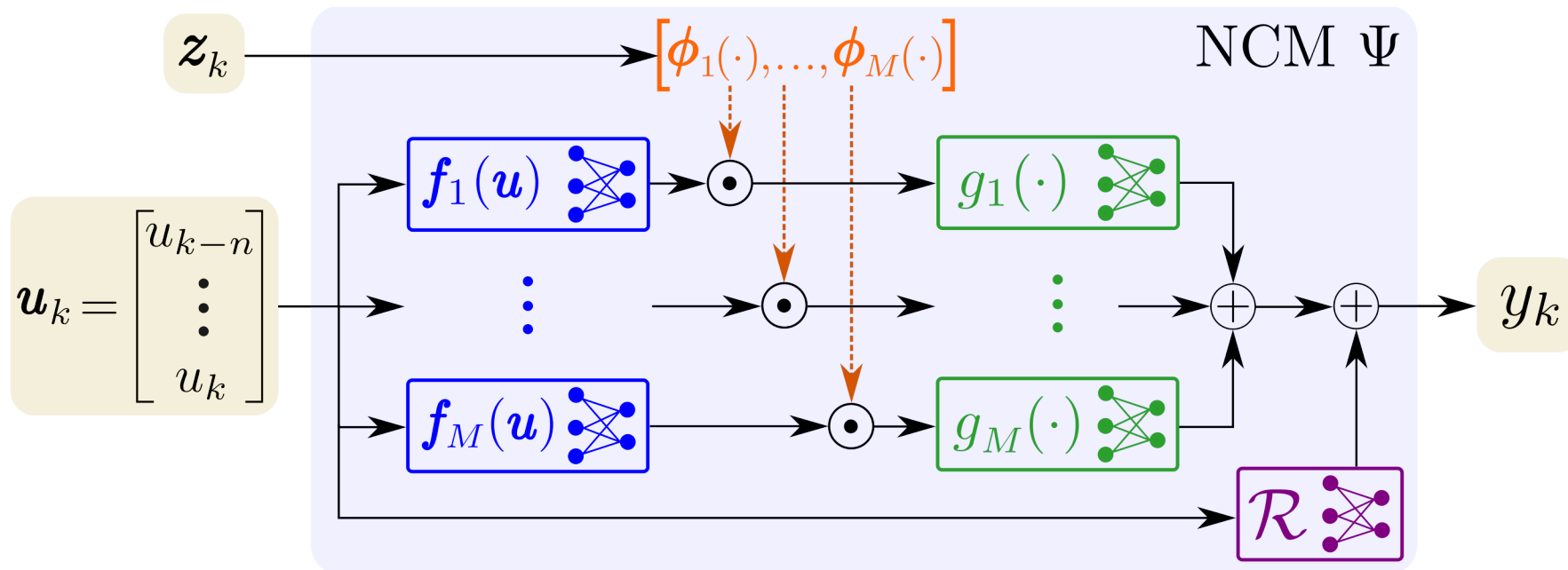


Neural Compositional Modules: Internal Architecture

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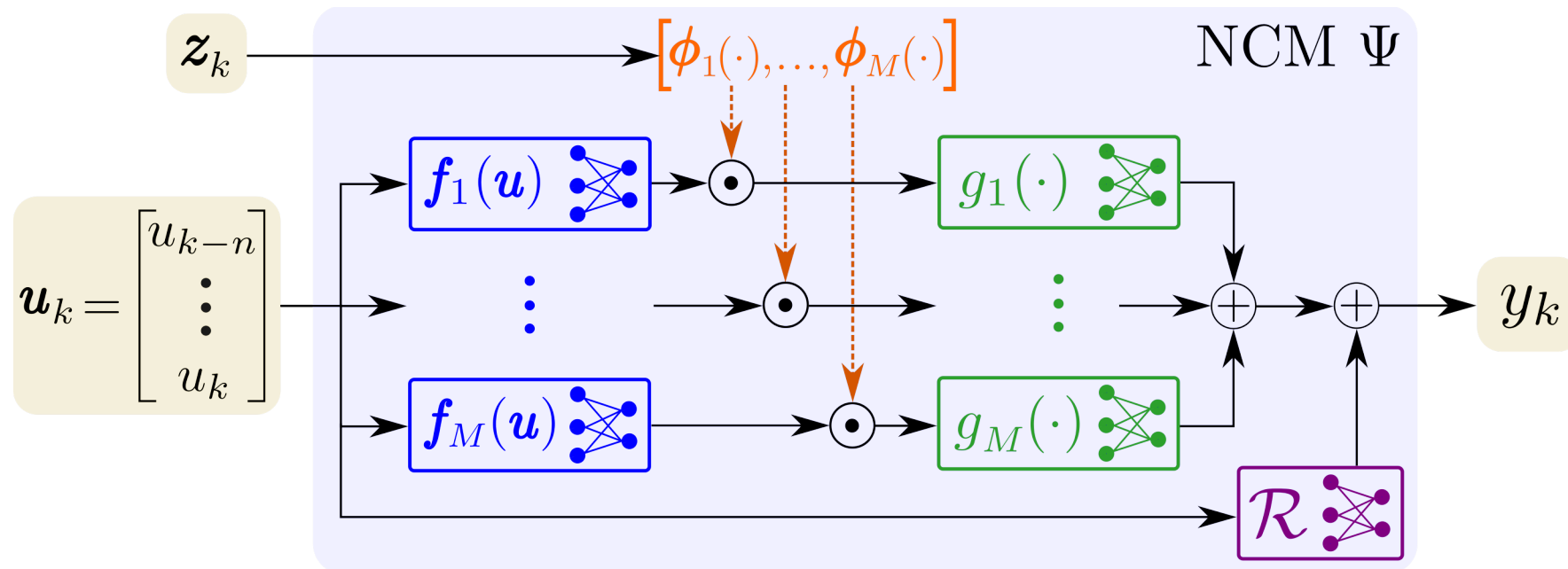
If the physical system is **fully nonlinear** :

$$y_k = \sum_{j=1}^M [\mathbf{f}(\mathbf{u}_k) \odot \boldsymbol{\phi}_j(\mathbf{z}_k)] \cdot \mathbf{g}_j + \underbrace{\mathcal{R}(\mathbf{u}_k)}_{\text{general-purpose NN (residual learning)}}$$



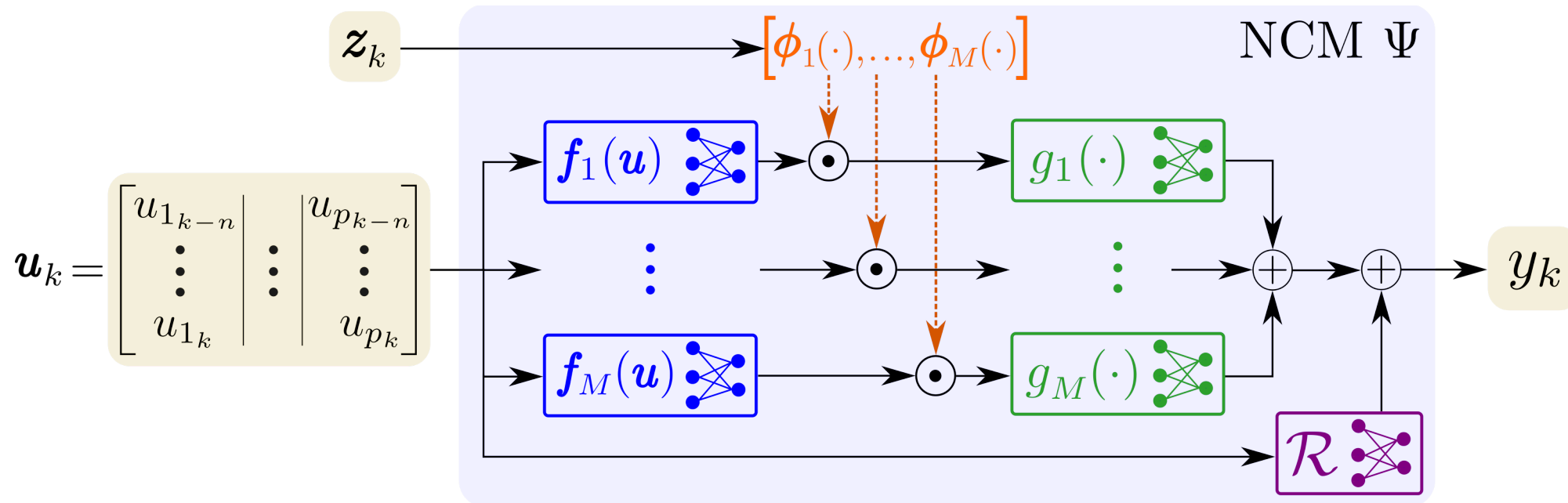
Neural Compositional Modules: Notes and Extensions

- The system can have p **inputs** (e.g., pedals and steering angle for a car)



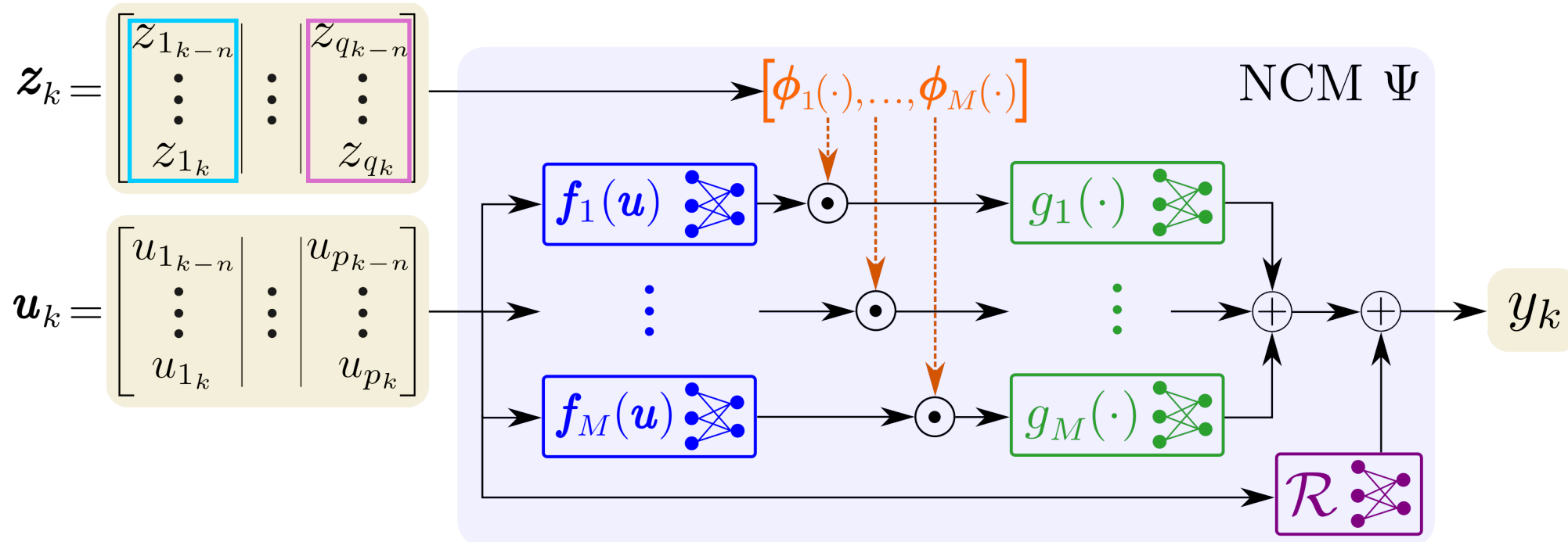
Neural Compositional Modules: Notes and Extensions

- The system can have p **inputs** (e.g., pedals and steering angle for a car)



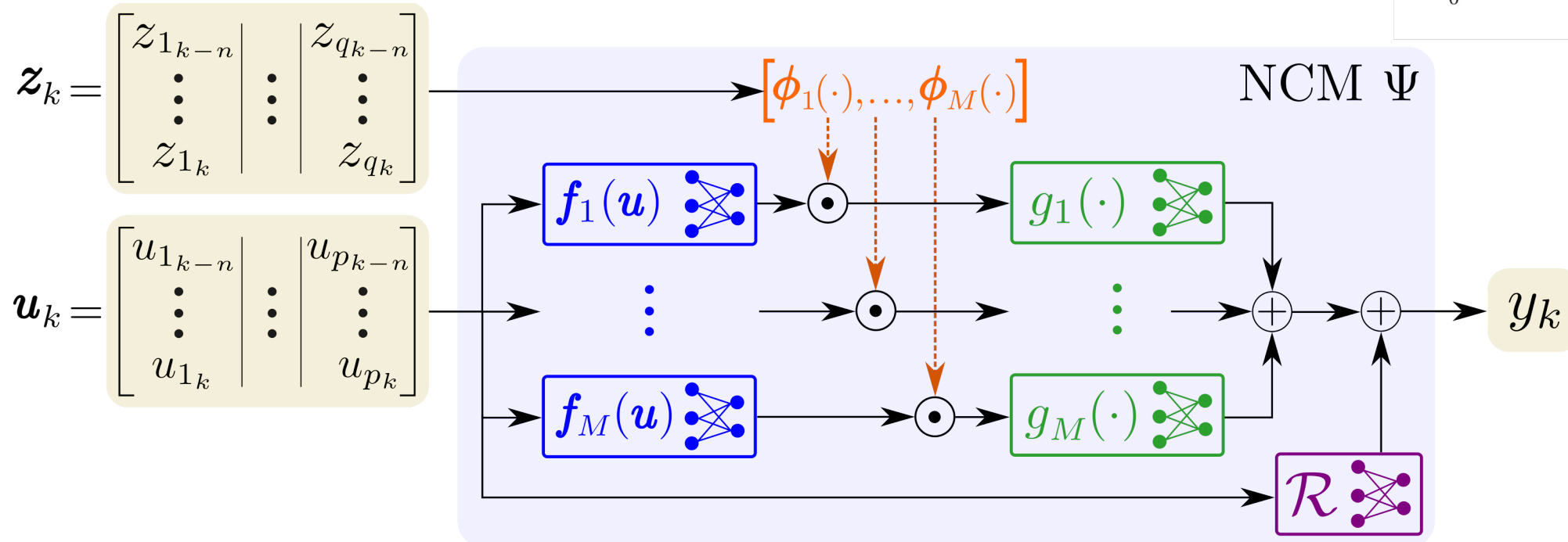
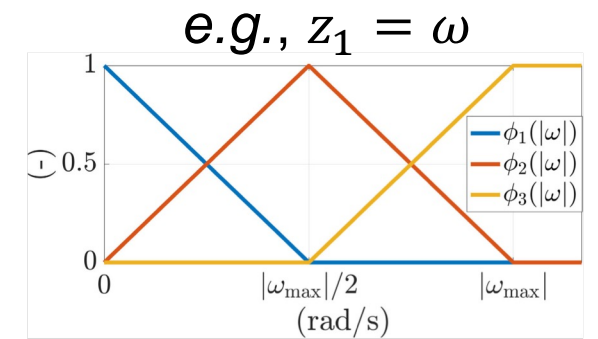
Neural Compositional Modules: Notes and Extensions

- The system can have p **inputs** (e.g., pedals and steering angle for a car)
- The system can have q **scheduling variables** $\mathbf{z}$ (e.g., **engine speed** and **gears**)
 - $\mathbf{z}_{1_k} = [z_{1_{k-n}}, \dots, z_{1_k}]$, $\mathbf{z}_{q_k} = [z_{q_{k-n}}, \dots, z_{q_k}]$, $\mathbf{z}_k = [\mathbf{z}_{1_k}, \dots, \mathbf{z}_{q_k}]$
 - $\phi_1(\mathbf{z}_k) = \phi_1(\mathbf{z}_{1_k}) \odot \dots \odot \phi_1(\mathbf{z}_{q_k})$ (with $\odot$ denoting the element-wise product)
 - $\phi_M(\mathbf{z}_k) = \phi_M(\mathbf{z}_{1_k}) \odot \dots \odot \phi_M(\mathbf{z}_{q_k})$ (with M denoting the n. of different operating regimes)



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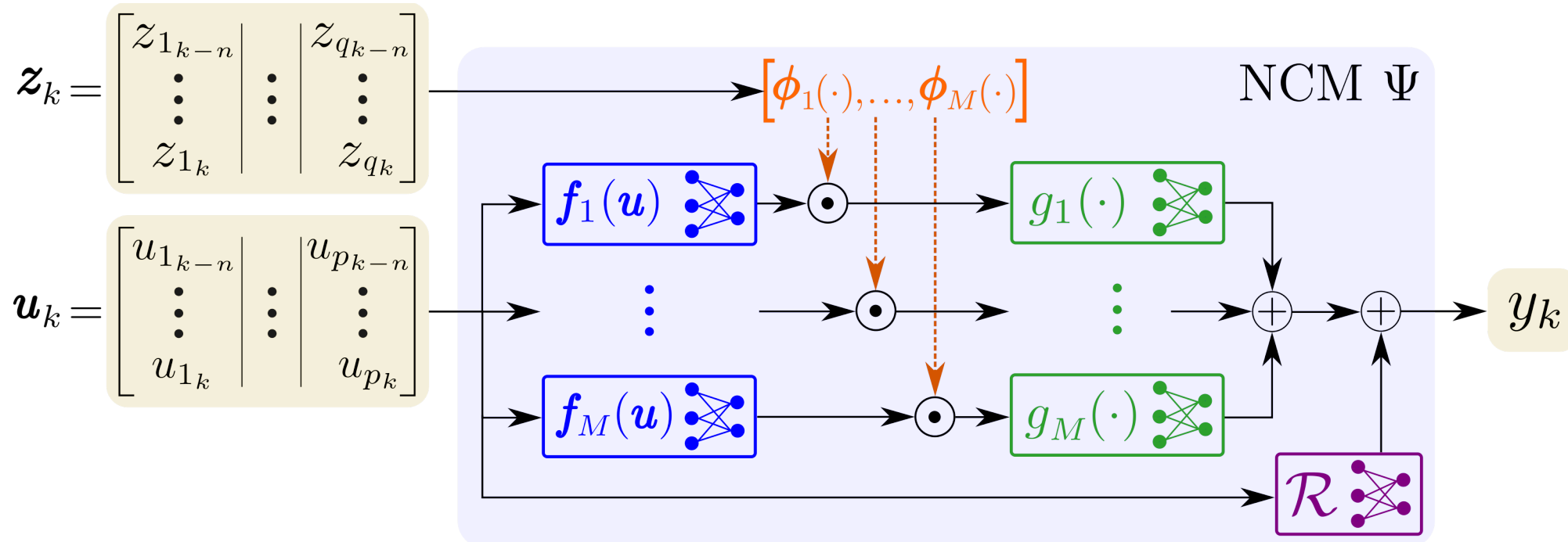
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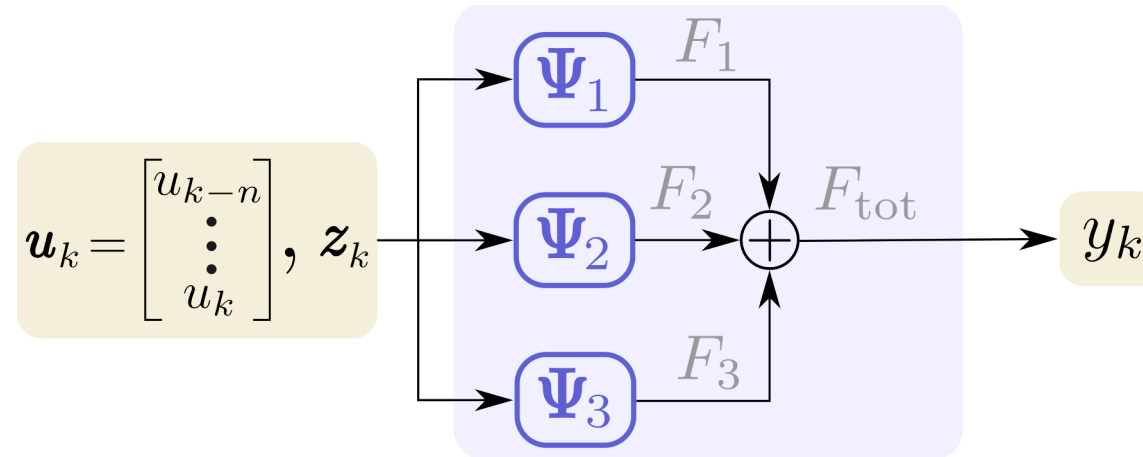
It still holds that $\sum_{j=1}^M \phi_j(\mathbf{z}_k) = 1$, $\phi_j(\mathbf{z}_k) \geq 0 \quad \forall \mathbf{z}_k$



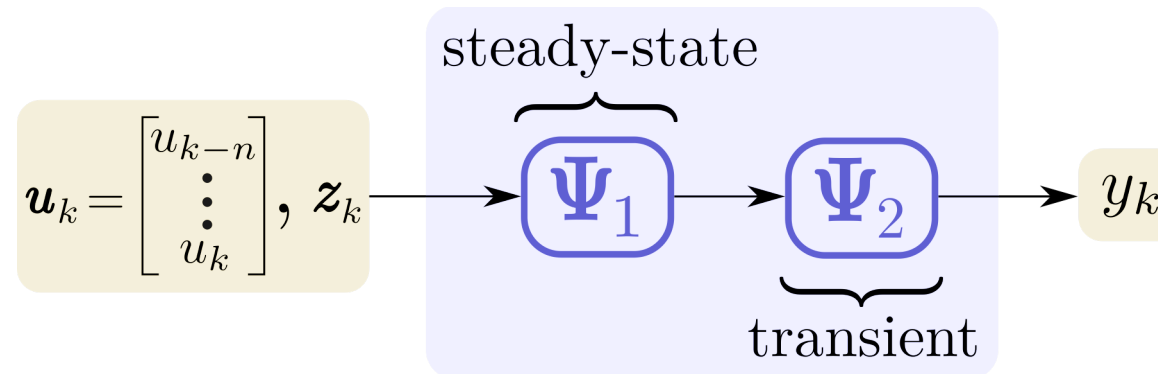
Why Combining Multiple Neural Compositional Modules?

Idea: Each NCM Ψ_i can be *specialized* to learn a sub-system / part of the dynamics

Example 1: Learning the effect of independent forces and moments



Example 2: Learning the steady-state and transient dynamics of a system



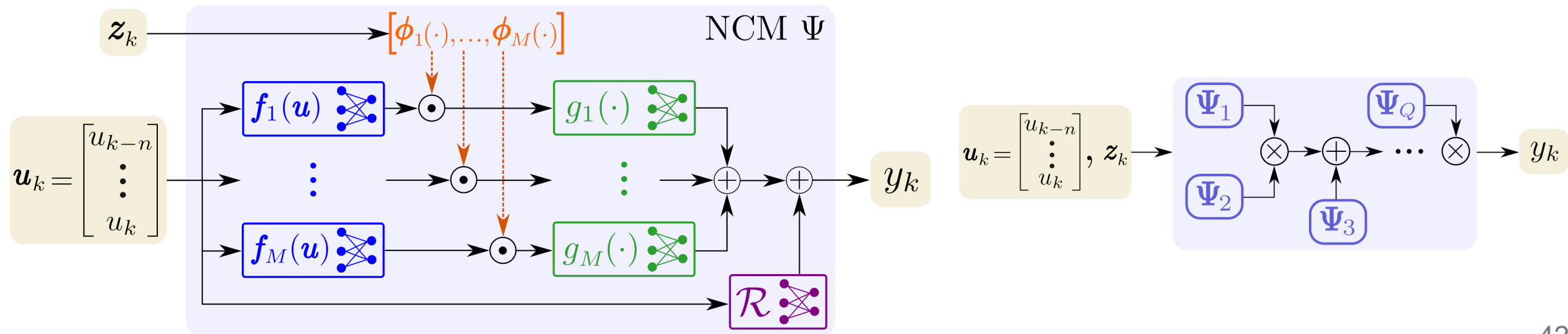
Pros and Cons of Neural Compositional Modules

Pros:

- The trained weights of the $f_j(\cdot)$ and $g_j(\cdot)$ may be physically interpreted
- Split task complexity in different NCMs Ψ
- Better generalization and sample efficiency than black-box ML models

Cons:

- Some design effort is still required in the $f_j(\cdot)$ modules



Questions & Discussion

